Spatial Frequencies, Sampling Functions, and Point Spread Functions

Fundamentals of Radio Interferometry (Sections 5.1, 5.2)

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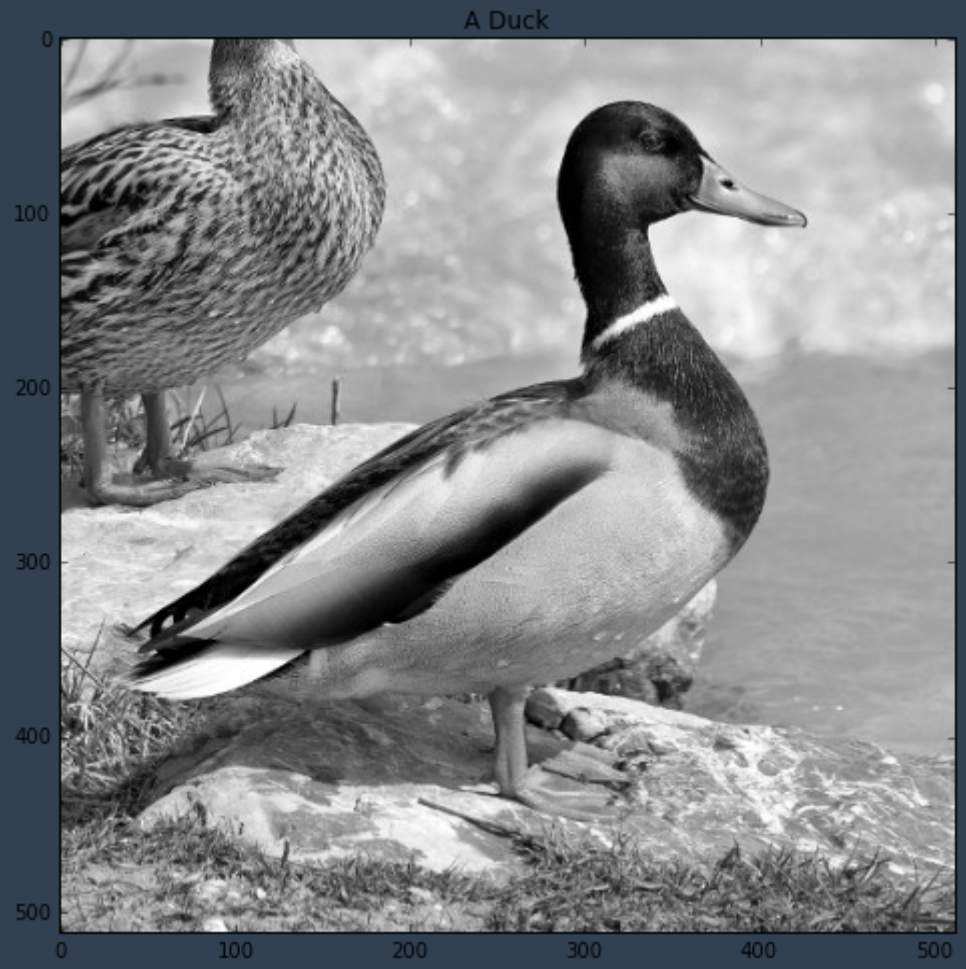
Dynamism of a Cyclist by Umberto Boccioni

A Duck



Spatial Transform of an Image

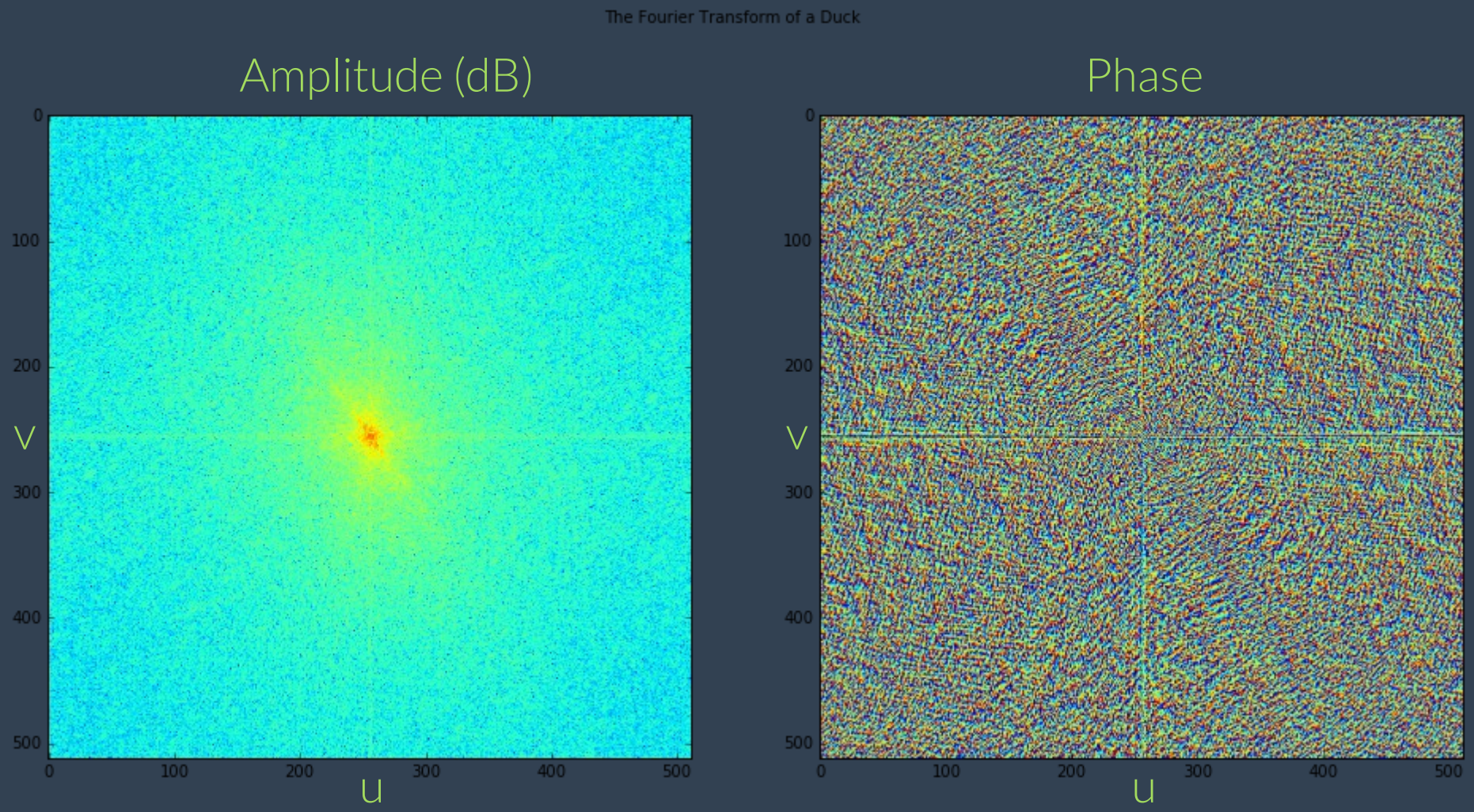
2-D Discrete Fourier Transform (using FFTs)

$$\mathcal{V} \rightleftharpoons I$$

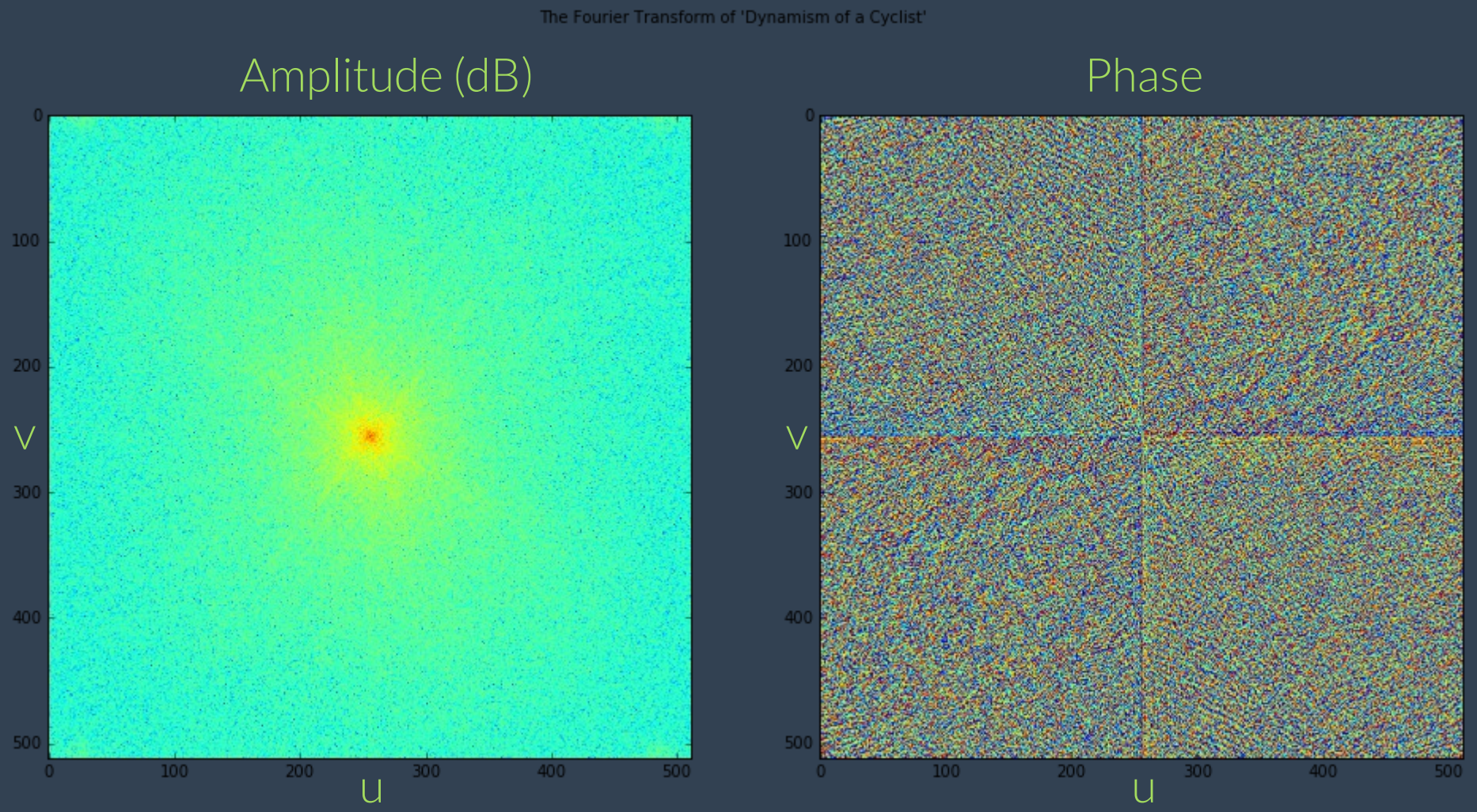
I : real, positive-valued 2-D array

$\mathcal{V}$: complex-valued 2-D array $z = re^{\imath\phi}$

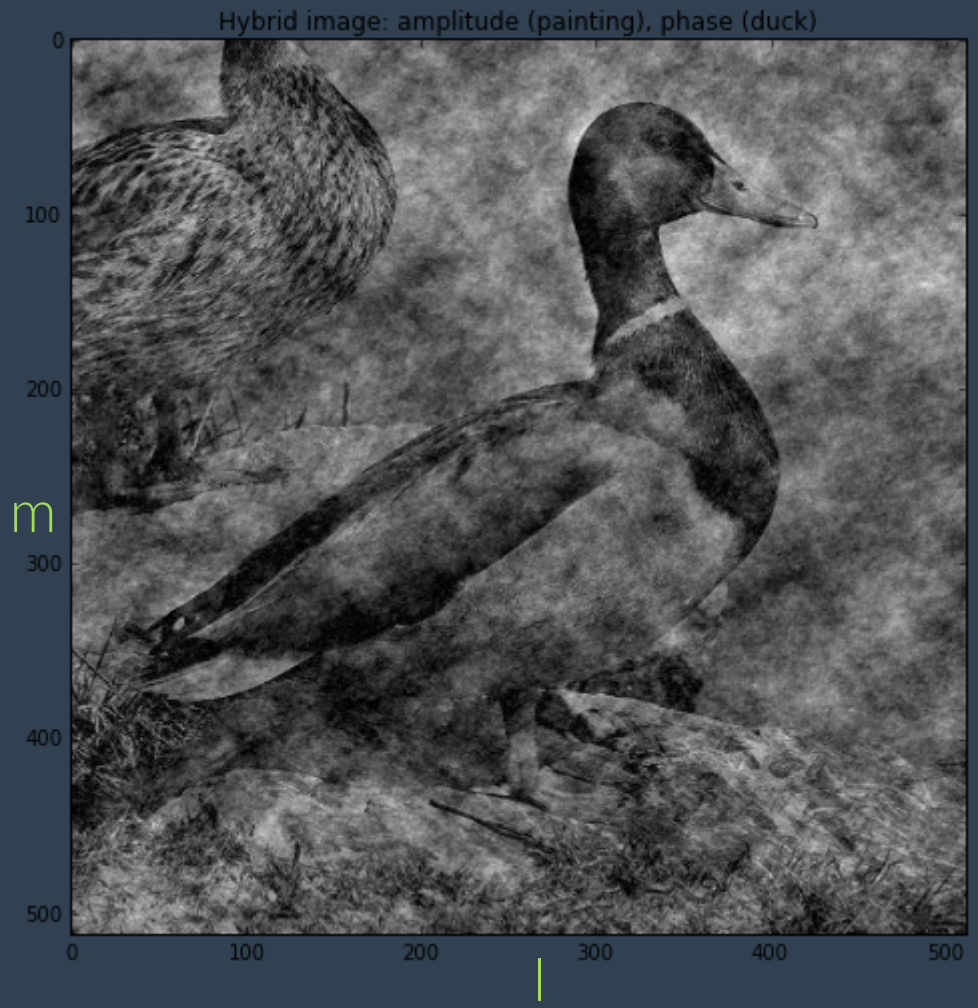
Fourier Transform(A Duck)



Fourier Transform(*Dynamism of a Cyclist*)

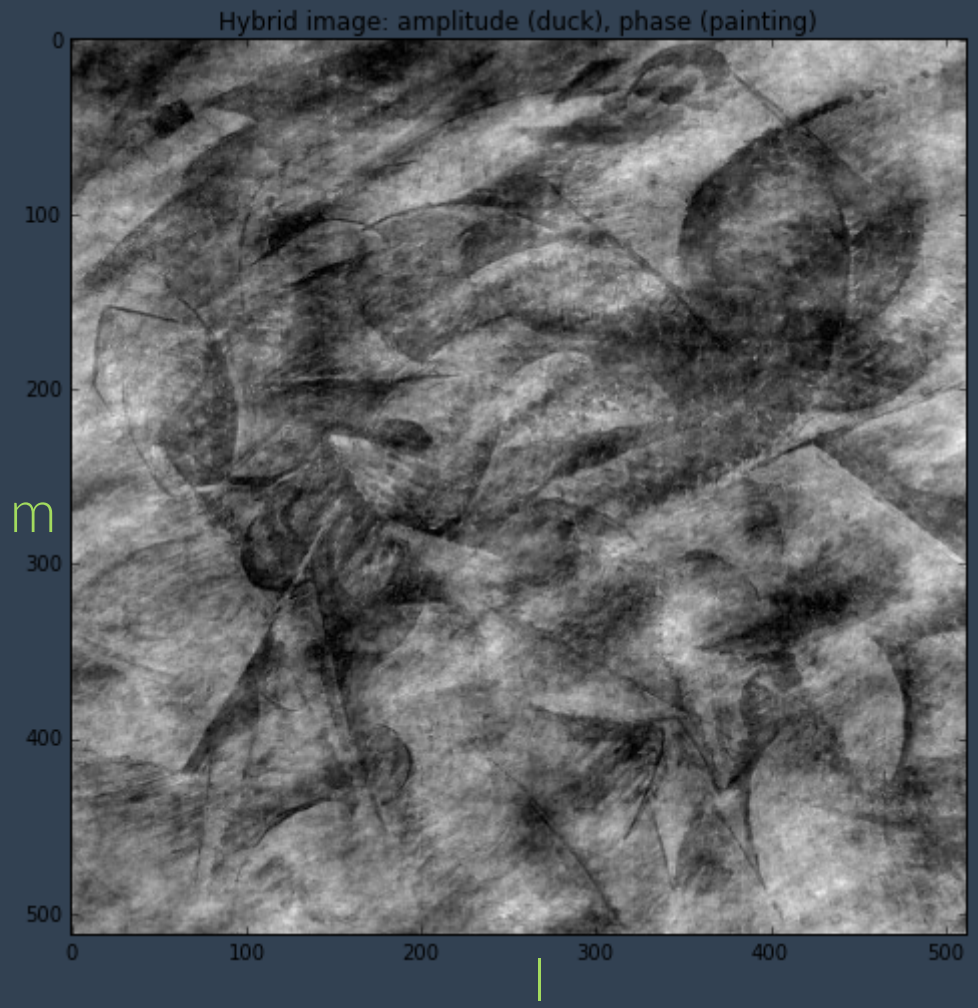


Hybrid [Amplitude (cyclist), Phase (duck)]



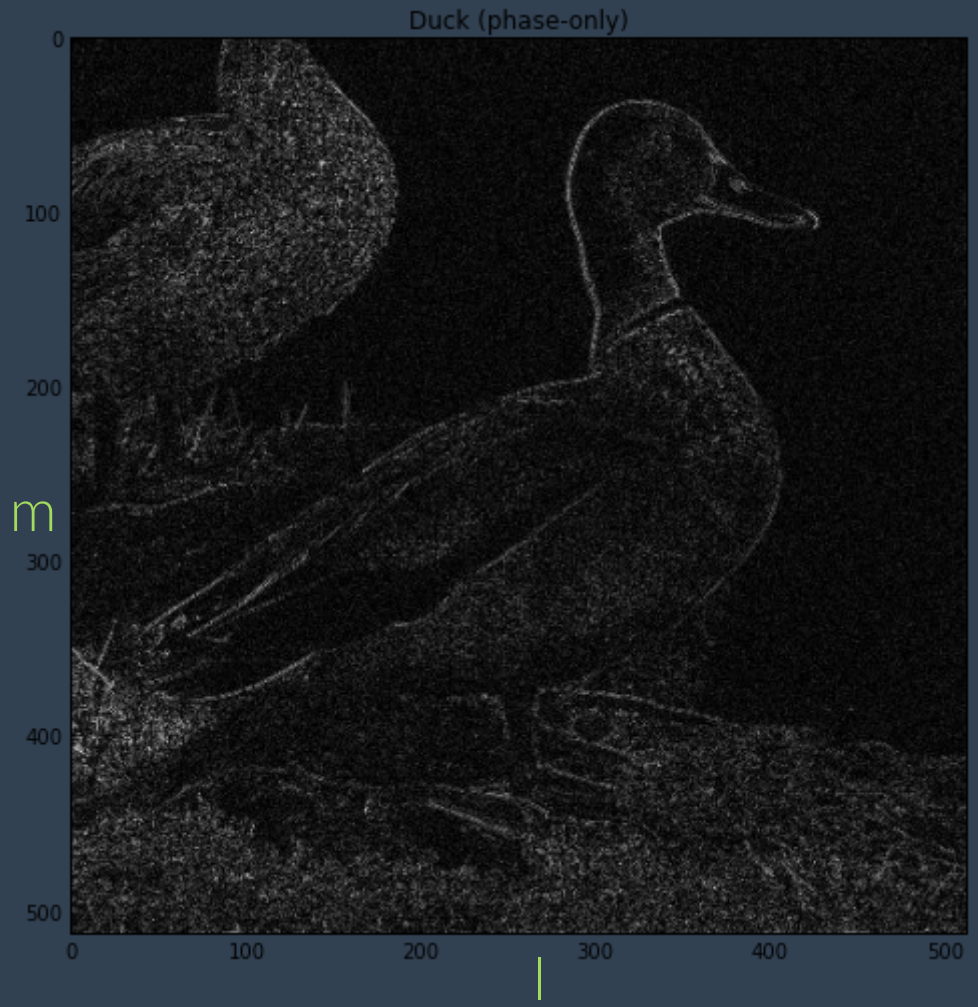
$$I_{\text{hybrid}} = \mathcal{F}\{r_{\text{cyclist}} e^{\imath \phi_{\text{duck}}}\}$$

Hybrid [Amplitude (duck), Phase (cyclist)]



$$I_{\text{hybrid}} = \mathcal{F}\{r_{\text{duck}} e^{\imath \phi_{\text{cyclist}}}\}$$

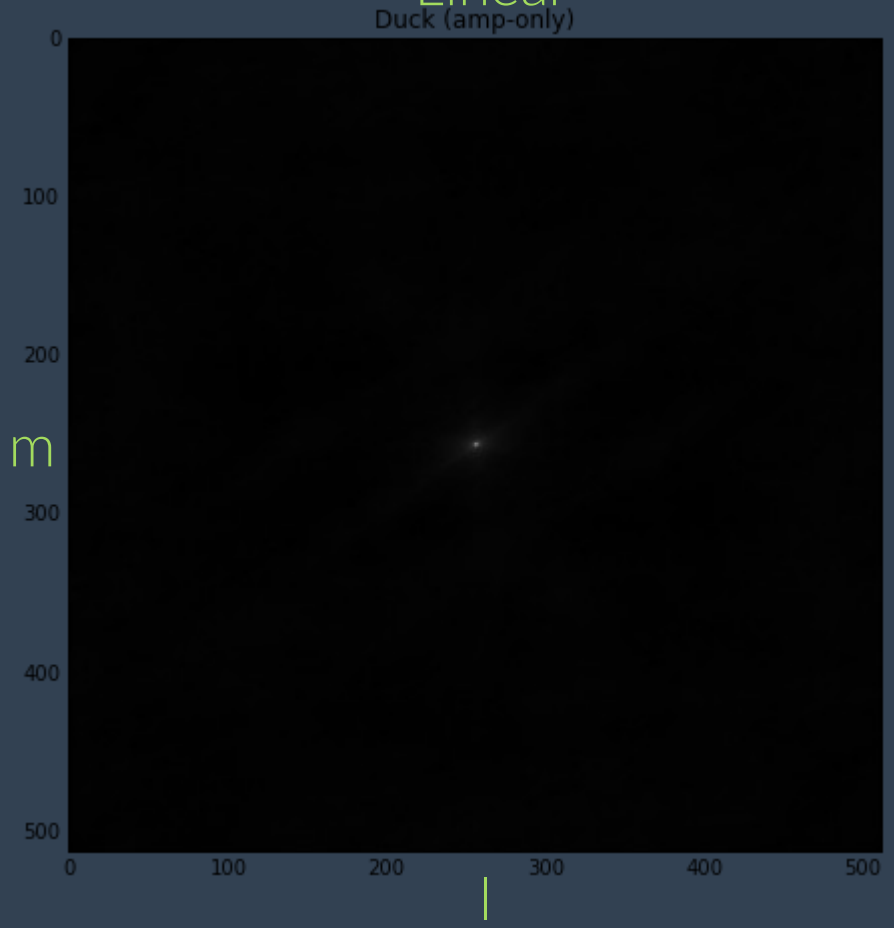
Phase-only Reconstruction



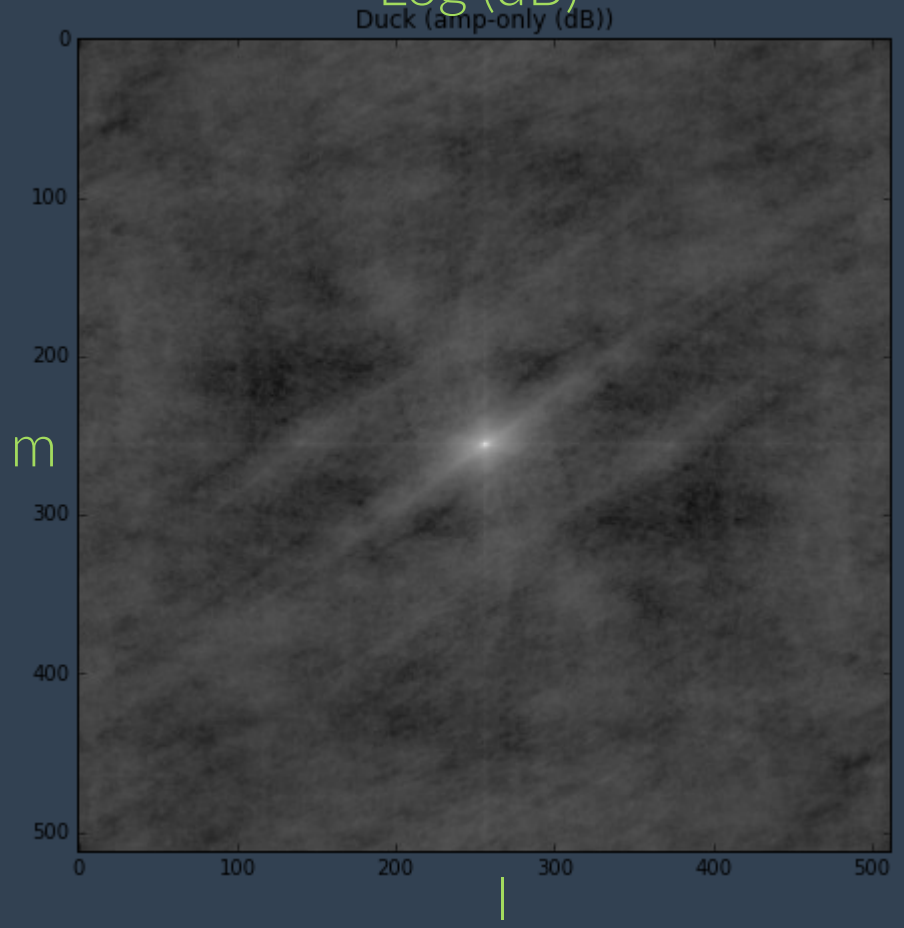
$$I_{\text{phase-only}} = \mathcal{F}\{1 \cdot e^{\imath\phi_{\text{duck}}}\}$$

Amplitude-only Reconstruction

Linear



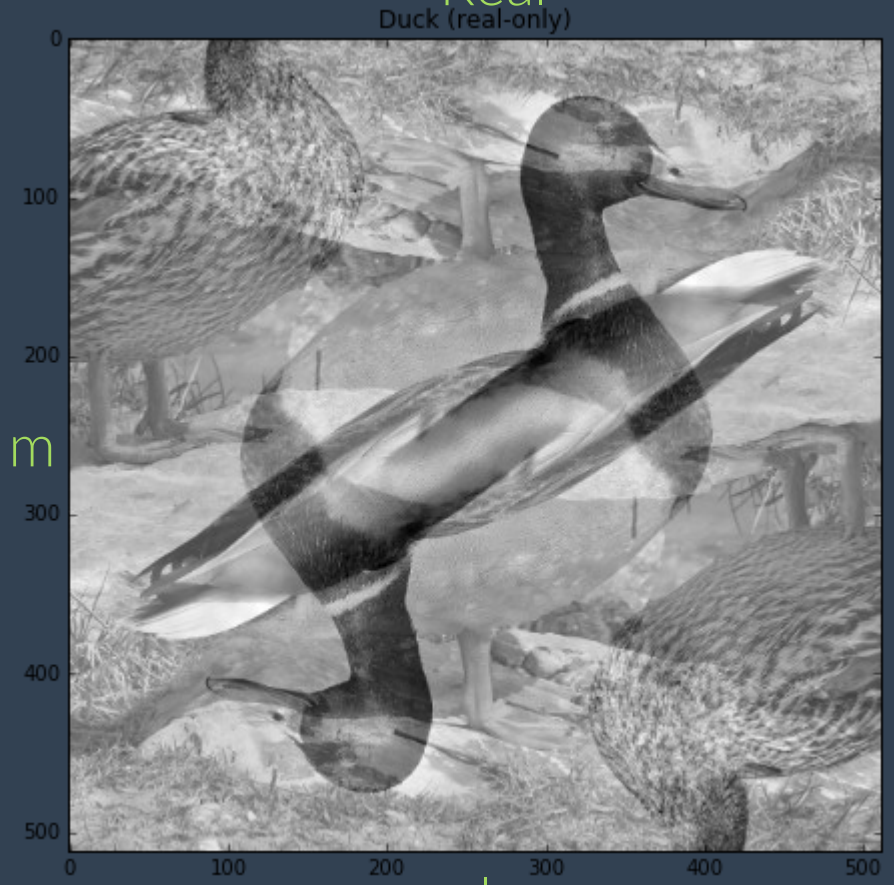
Log (dB)



$$I_{\text{amp-only}} = \mathcal{F}\{r_{\text{duck}} e^{i \cdot 0}\}$$

Complex Component Reconstruction

Real



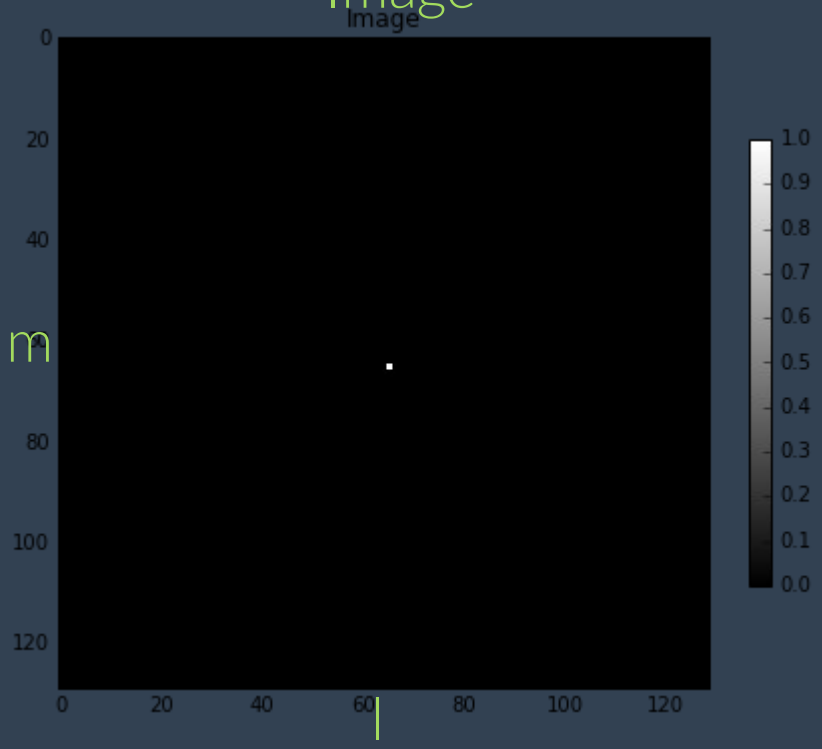
$$I_{\text{real}} = \mathcal{F}\{r\cos\phi\}$$

Imaginary

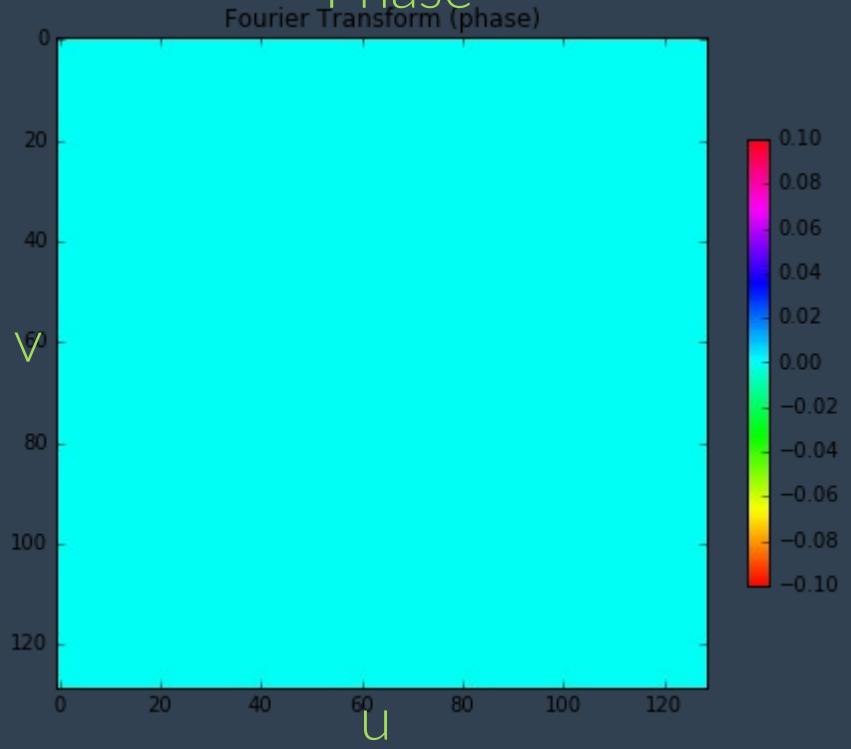
$$I_{\text{imag}} = \mathcal{F}\{ri\sin\phi\}$$

FT(Point Source) [at the Origin]

Image

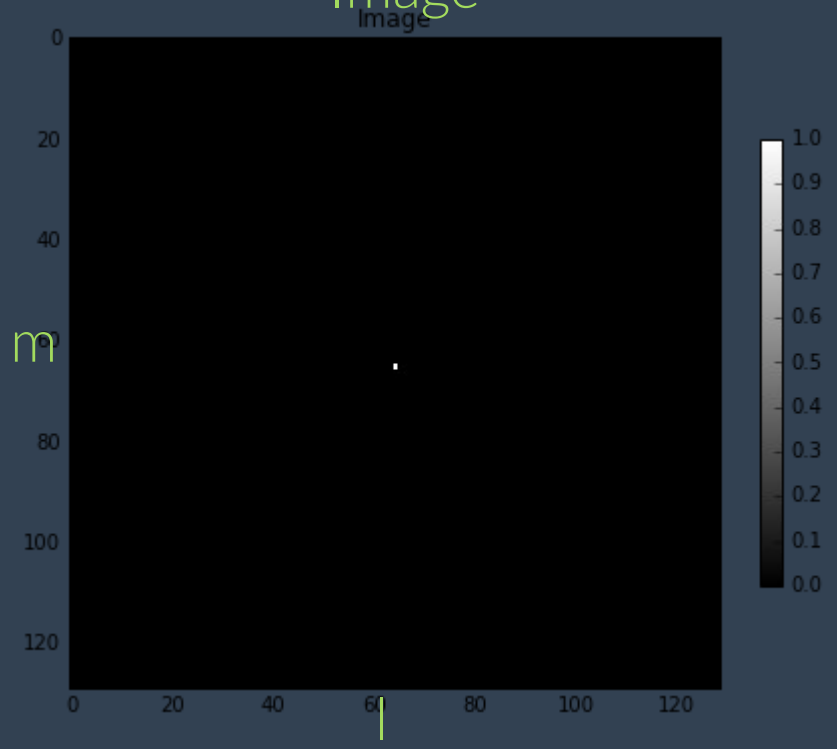


Phase



FT(Point Source) [1 pixel right of origin]

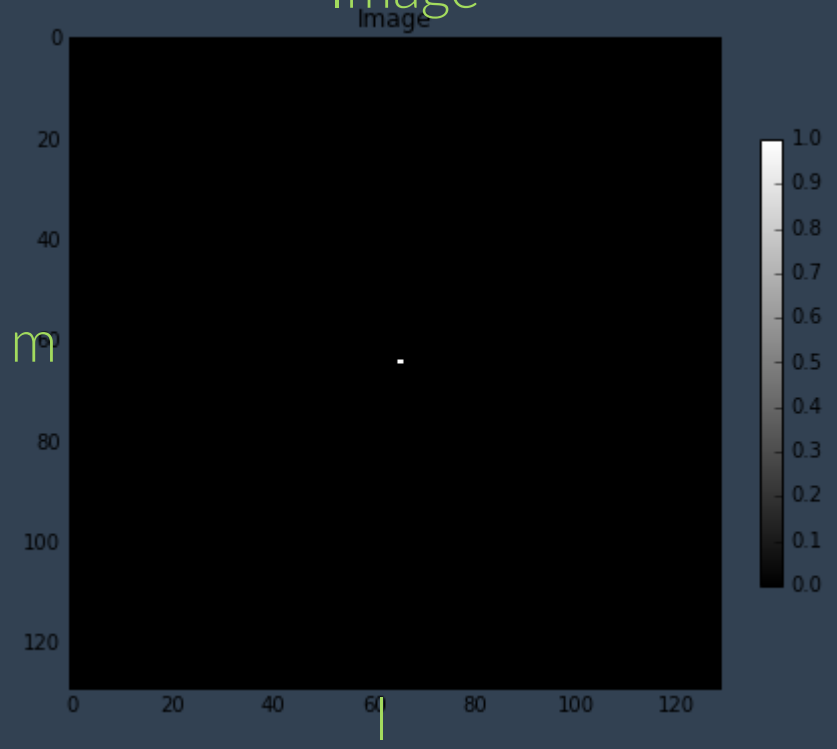
Image



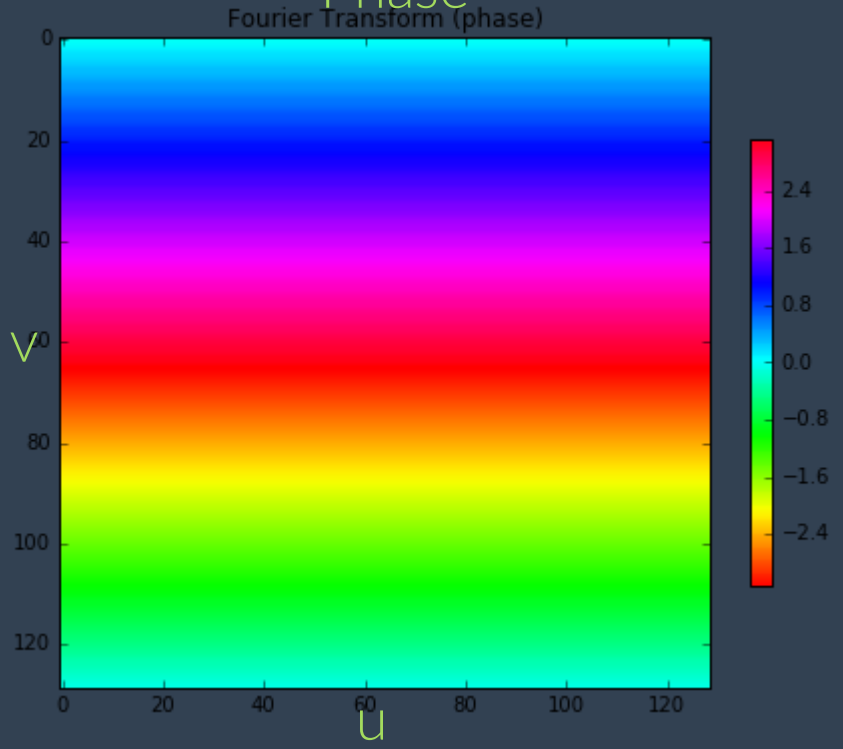
Phase

FT(Point Source) [1 pixel down from origin]

Image

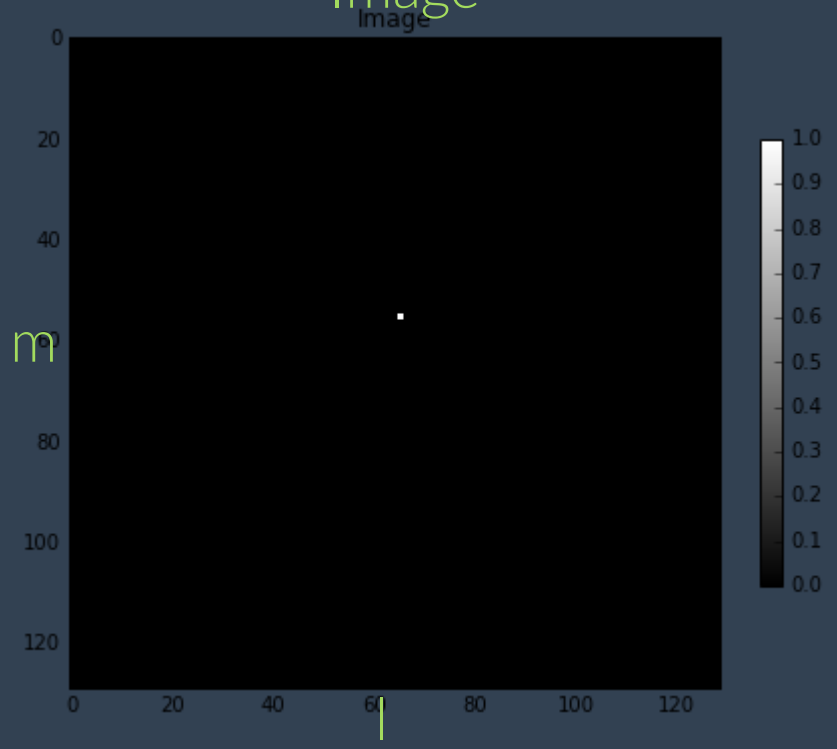


Phase

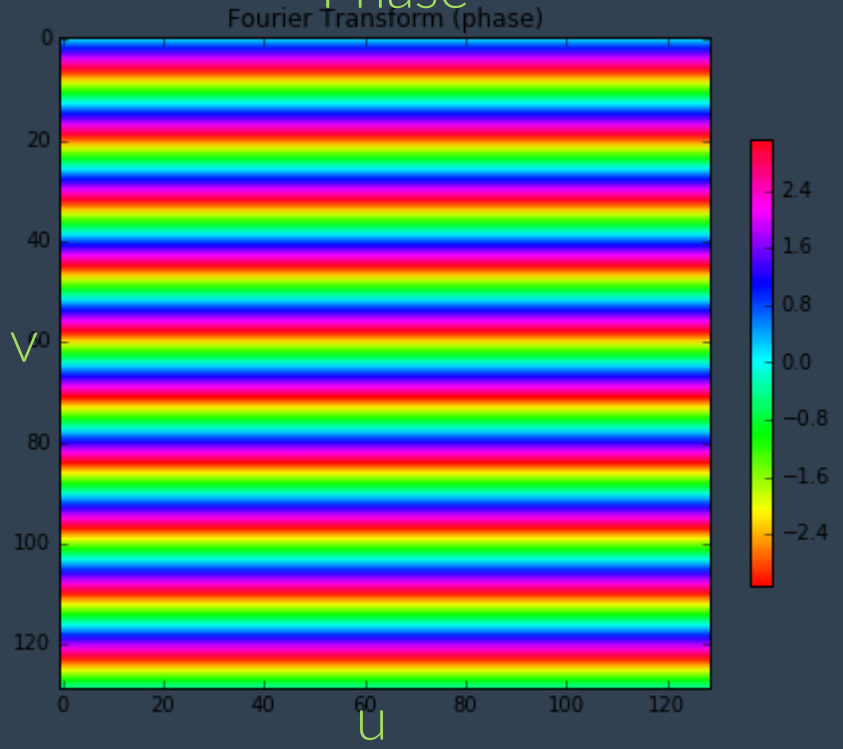


FT(Point Source) [10 pixels up from origin]

Image

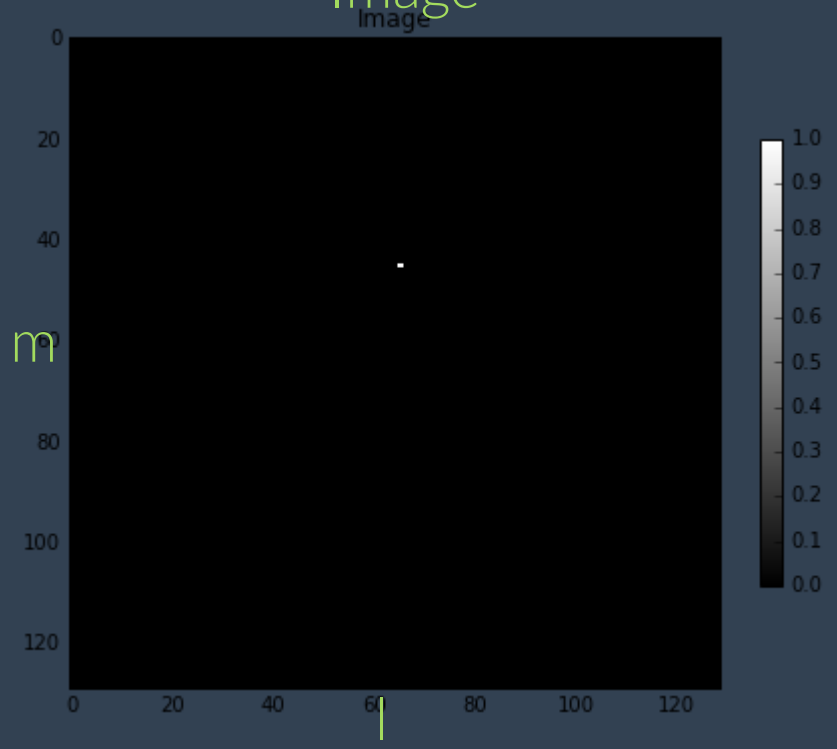


Phase

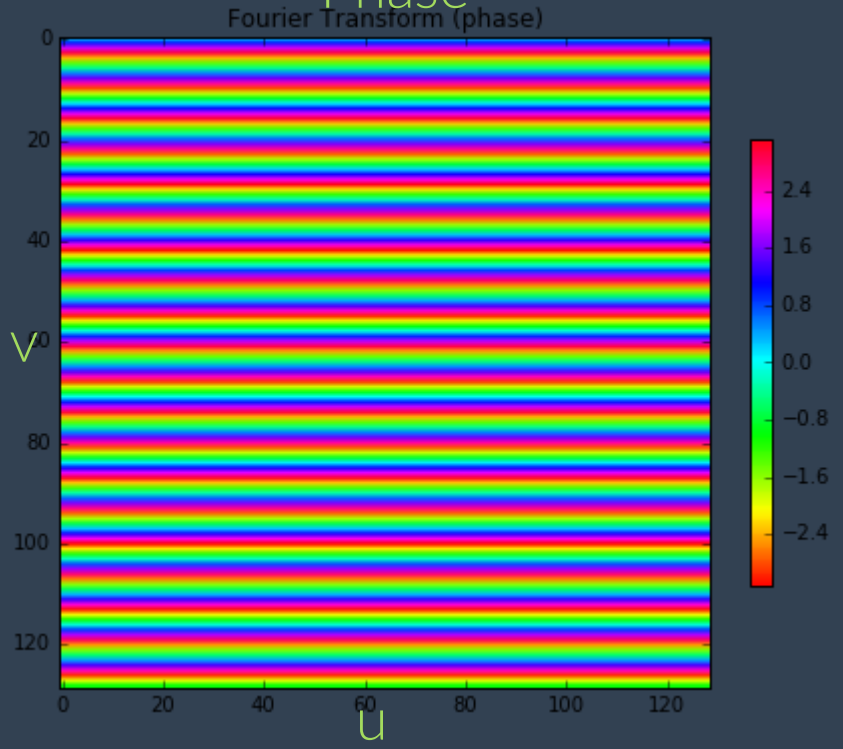


FT(Point Source) [20 pixels up from origin]

Image

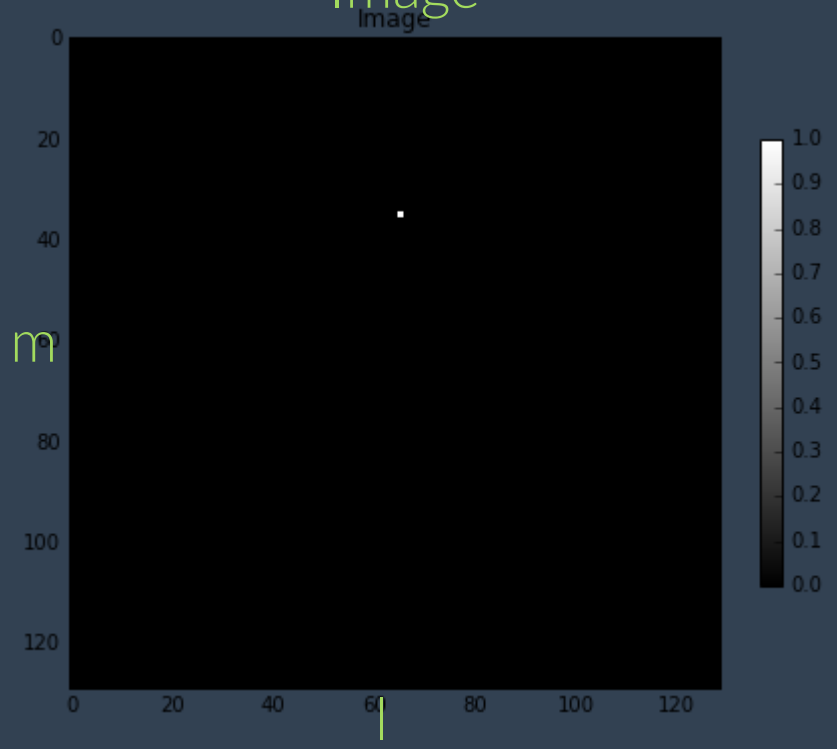


Phase

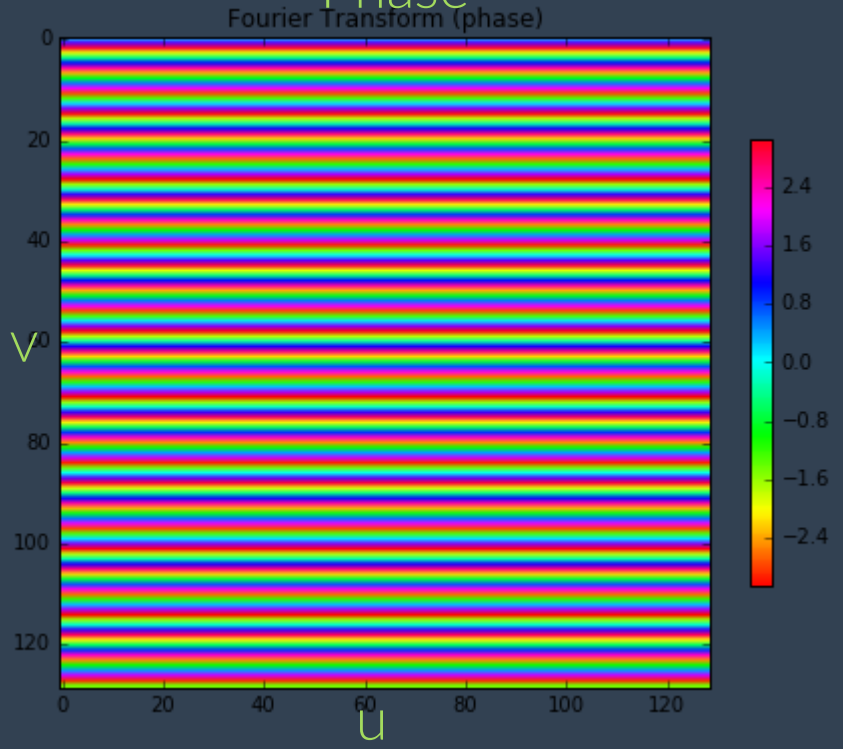


FT(Point Source) [30 pixels up from origin]

Image

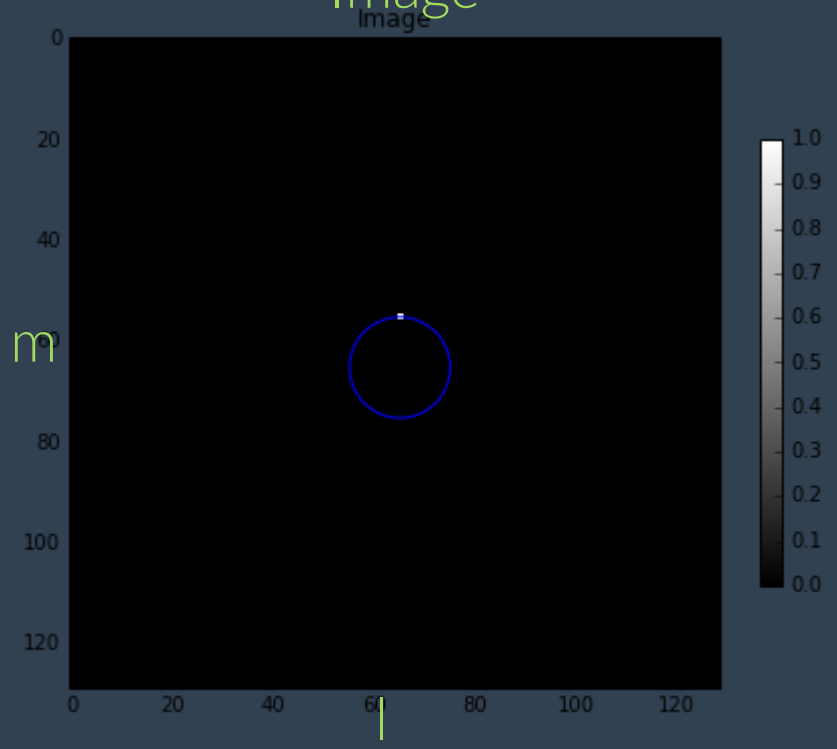


Phase

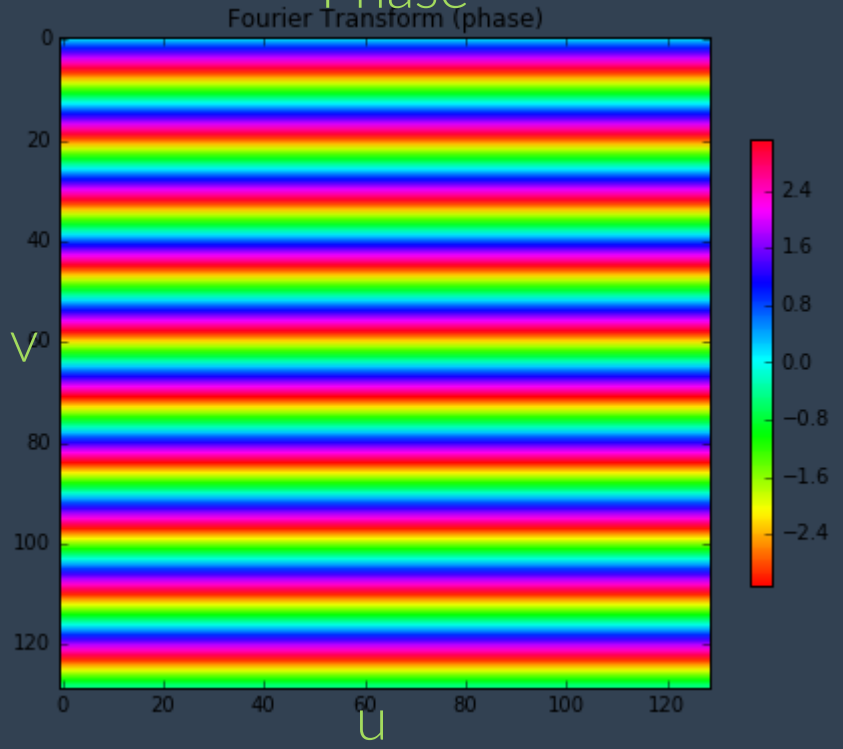


FT(Point Source) [rotated around circle]

Image

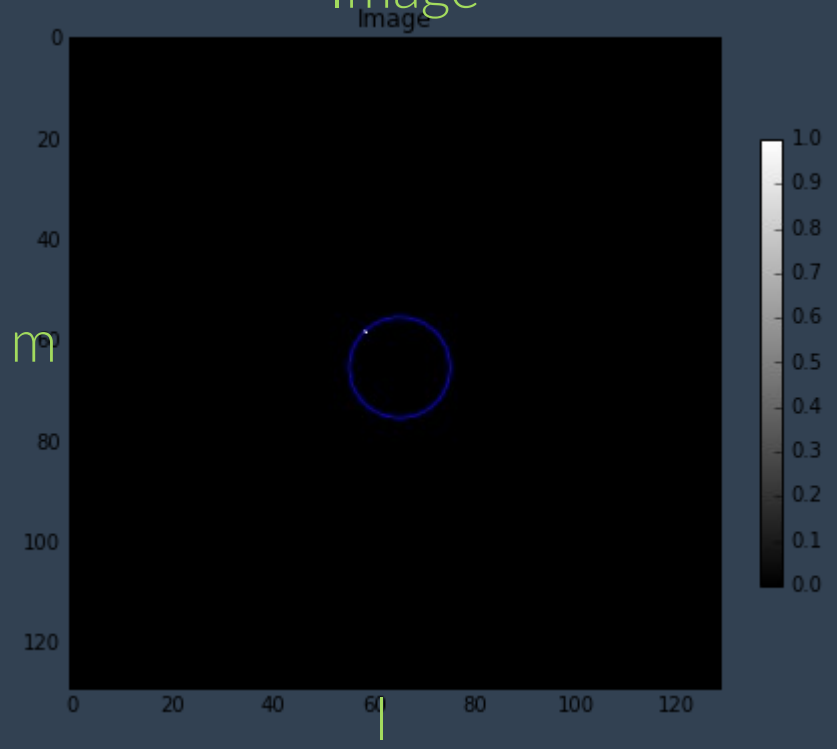


Phase

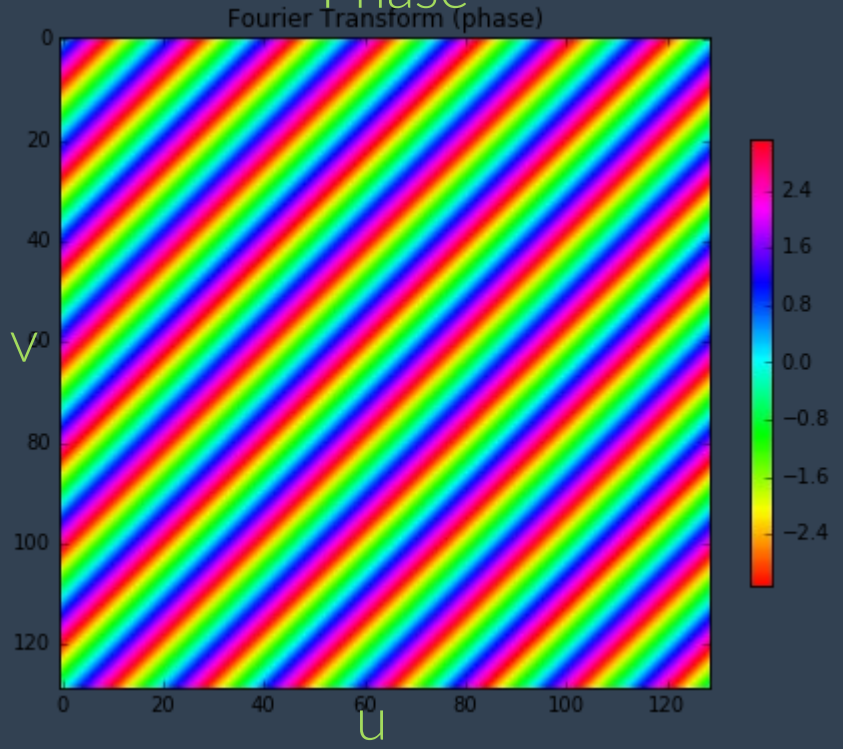


FT(Point Source) [rotated around circle]

Image

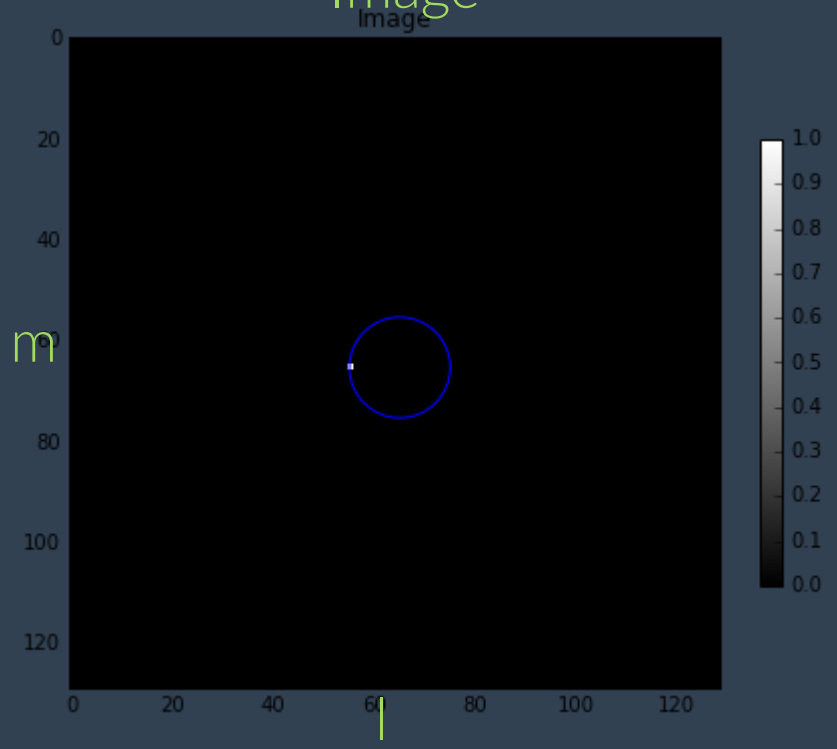


Phase

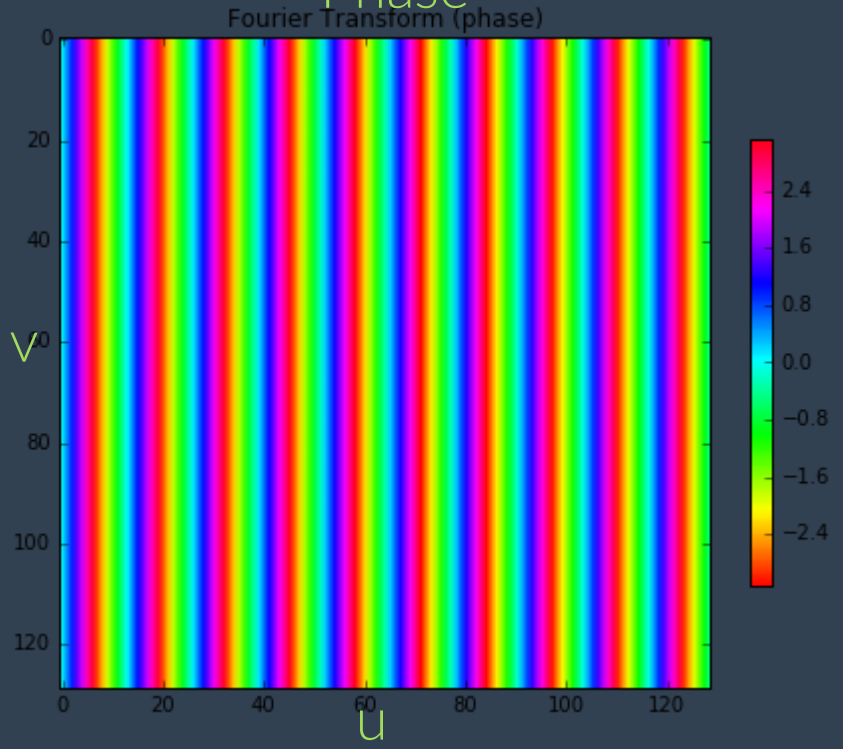


FT(Point Source) [rotated around circle]

Image

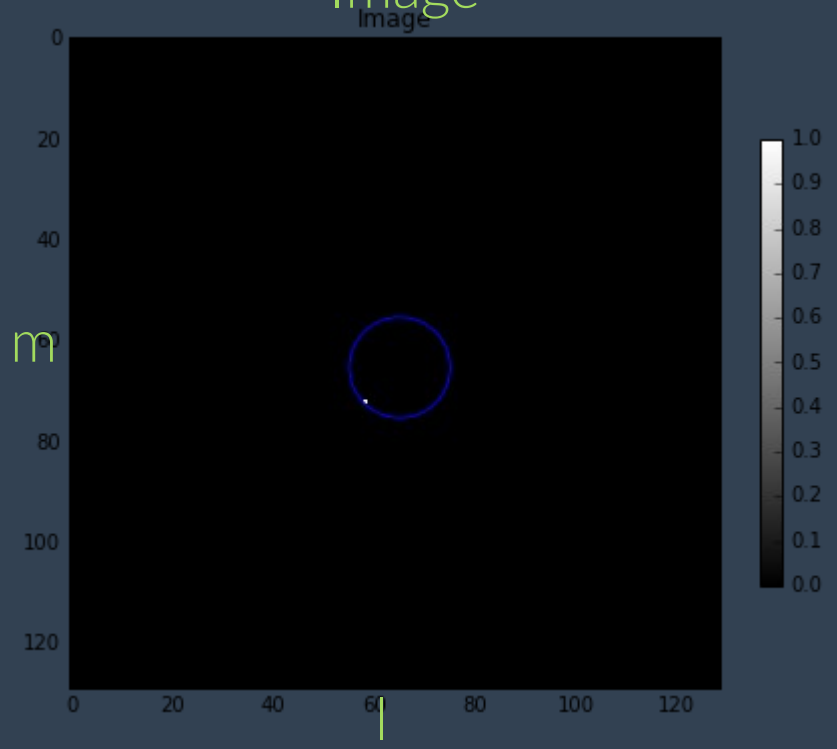


Phase

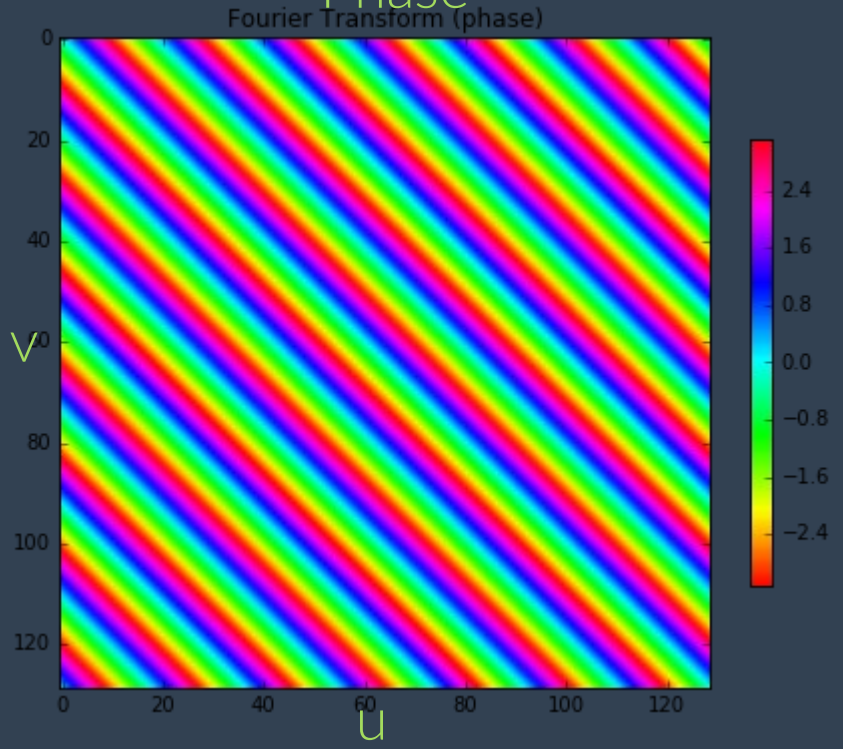


FT(Point Source) [rotated around circle]

Image

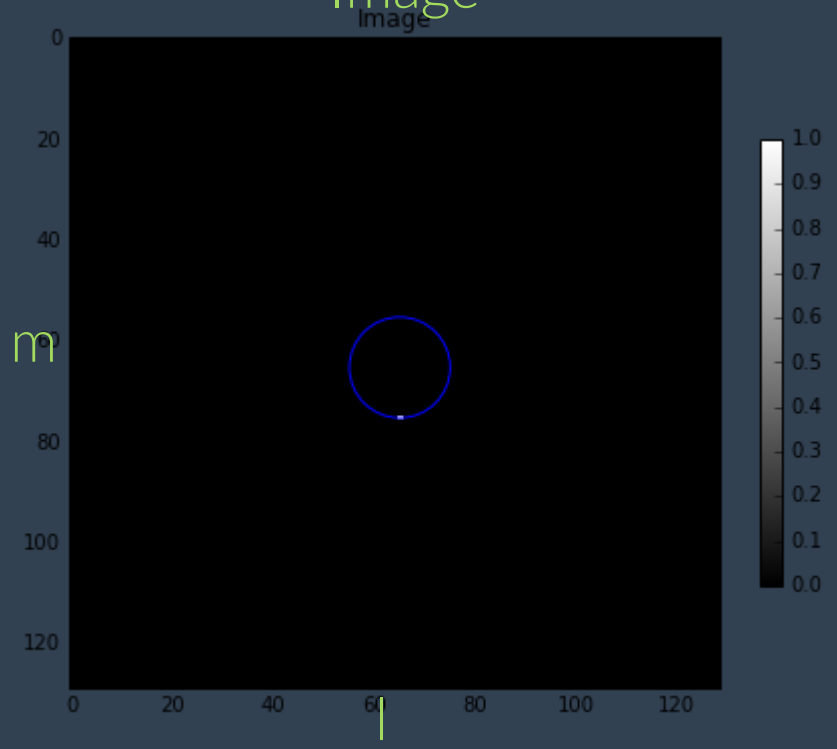


Phase



FT(Point Source) [rotated around circle]

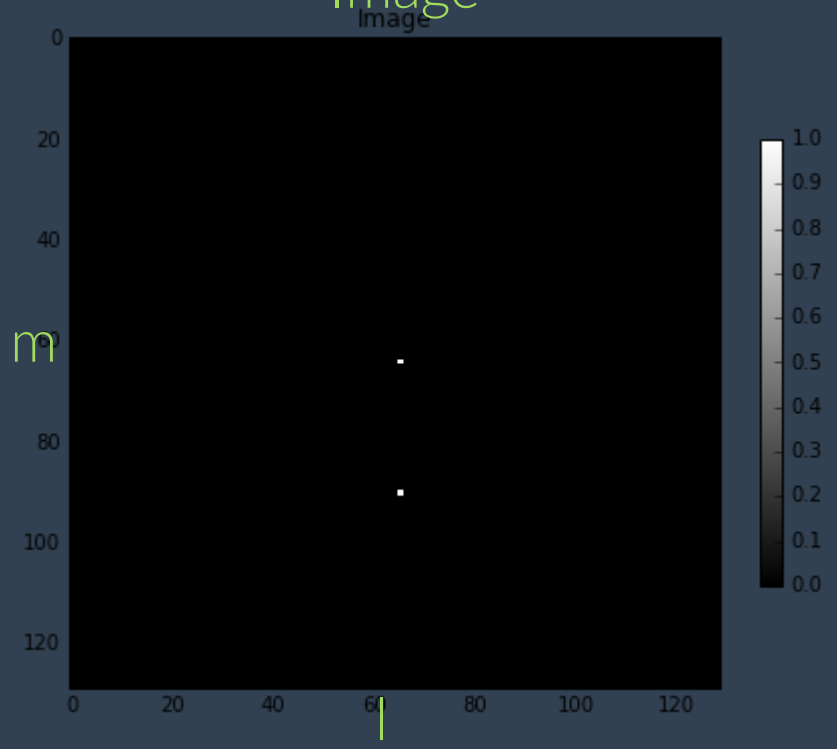
Image



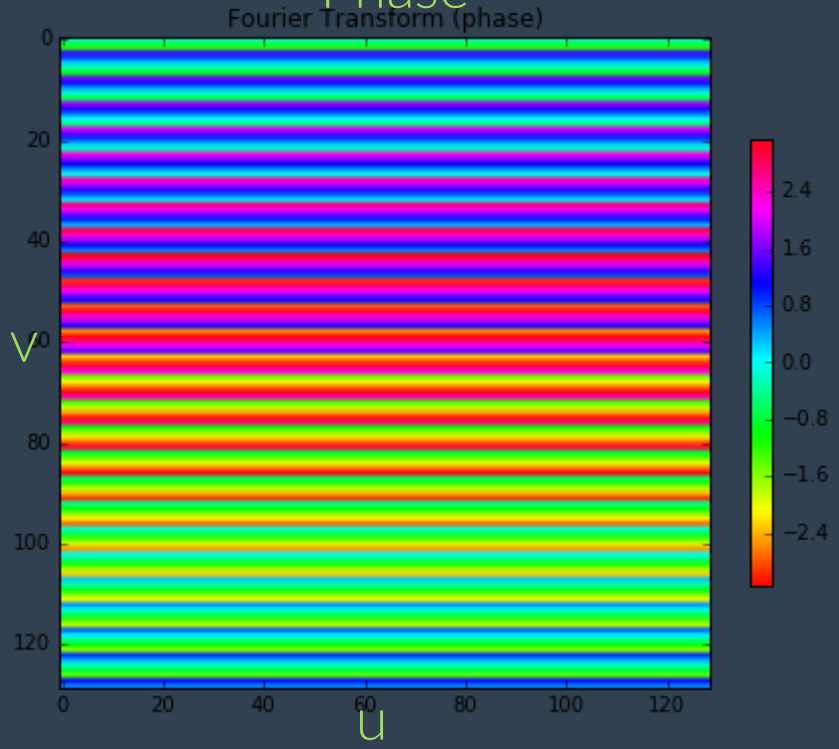
Phase

FT(2 Point Sources) [equal amplitude]

Image

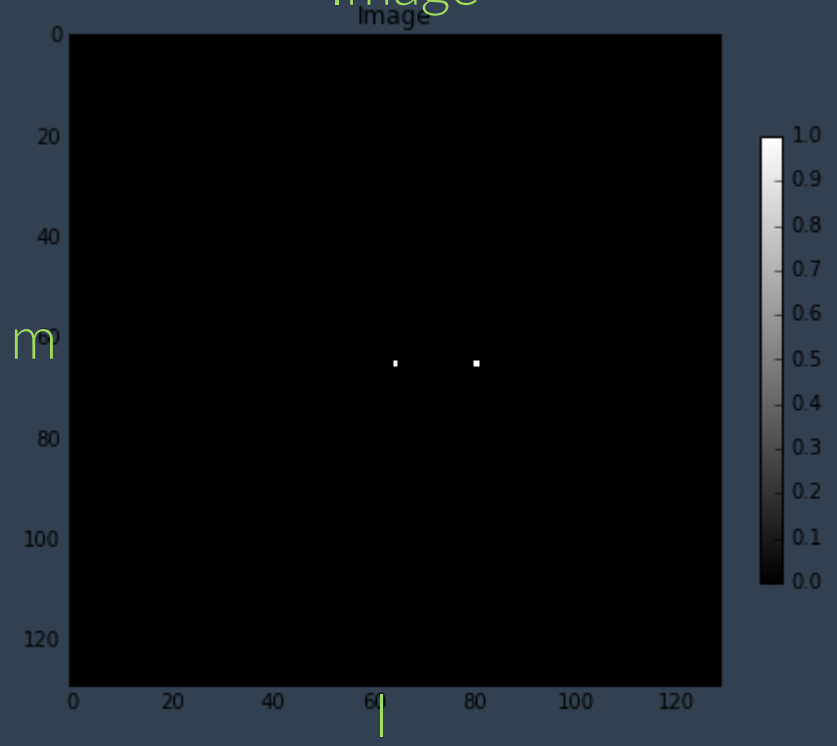


Phase

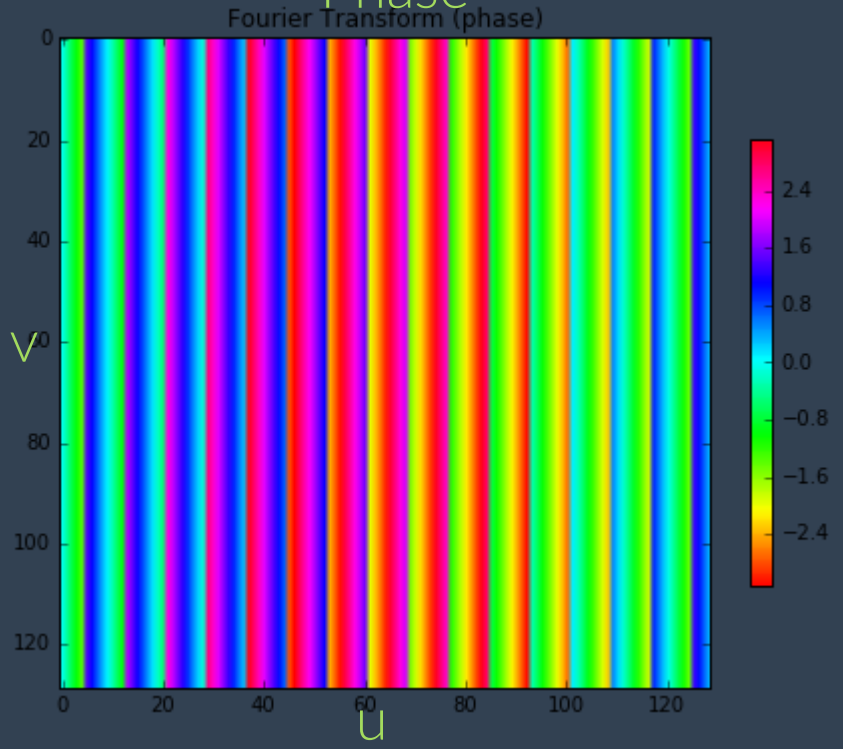


FT(2 Point Sources) [equal amplitude]

Image

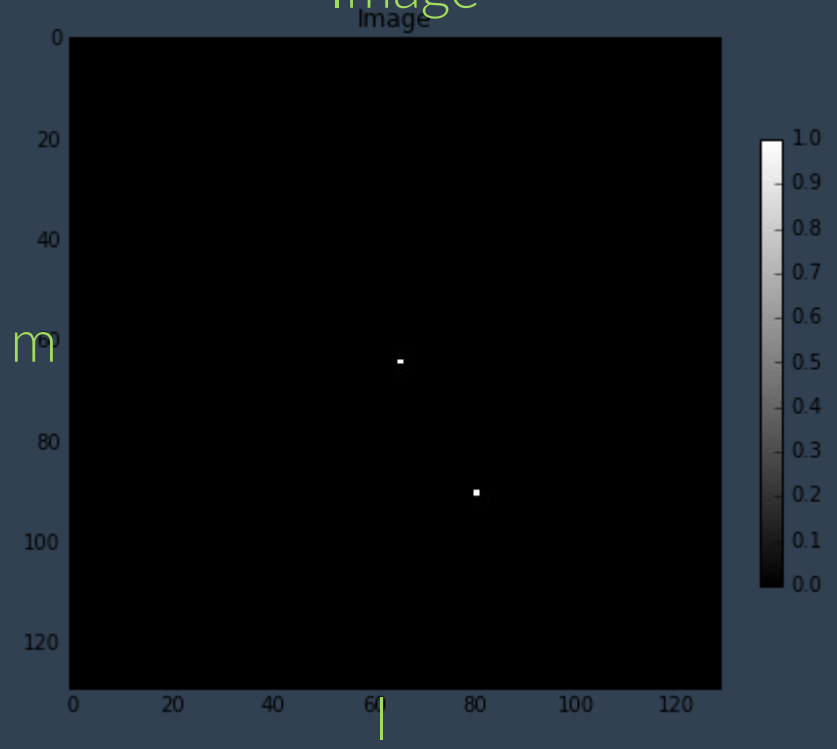


Phase

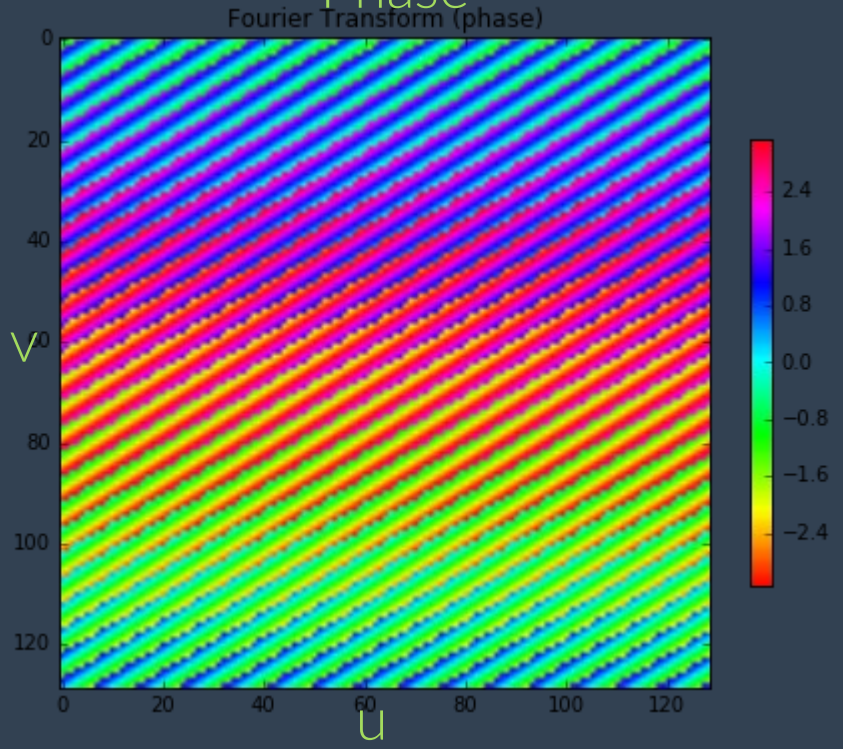


FT(2 Point Sources) [equal amplitude]

Image

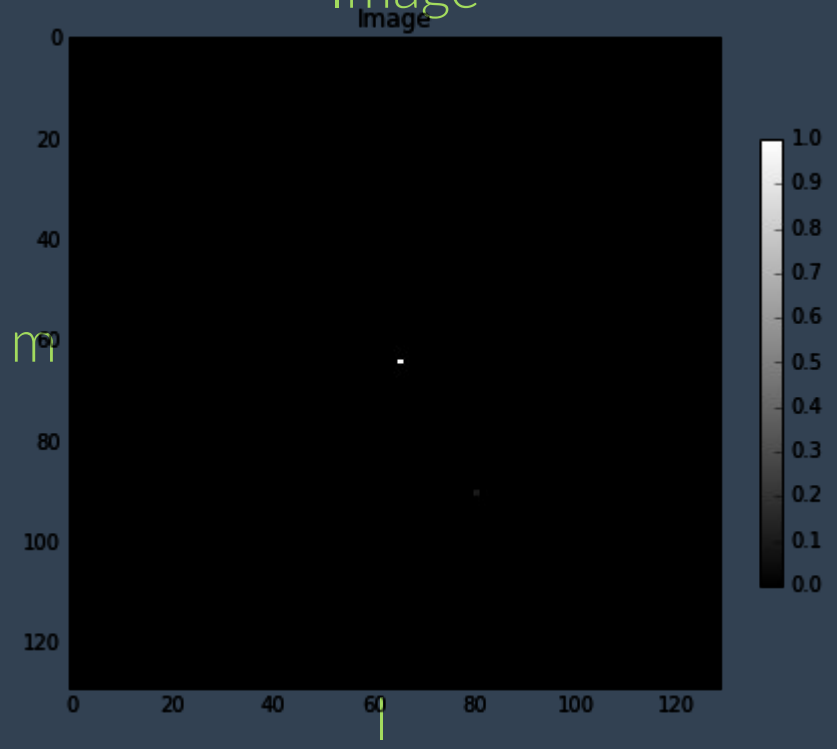


Phase

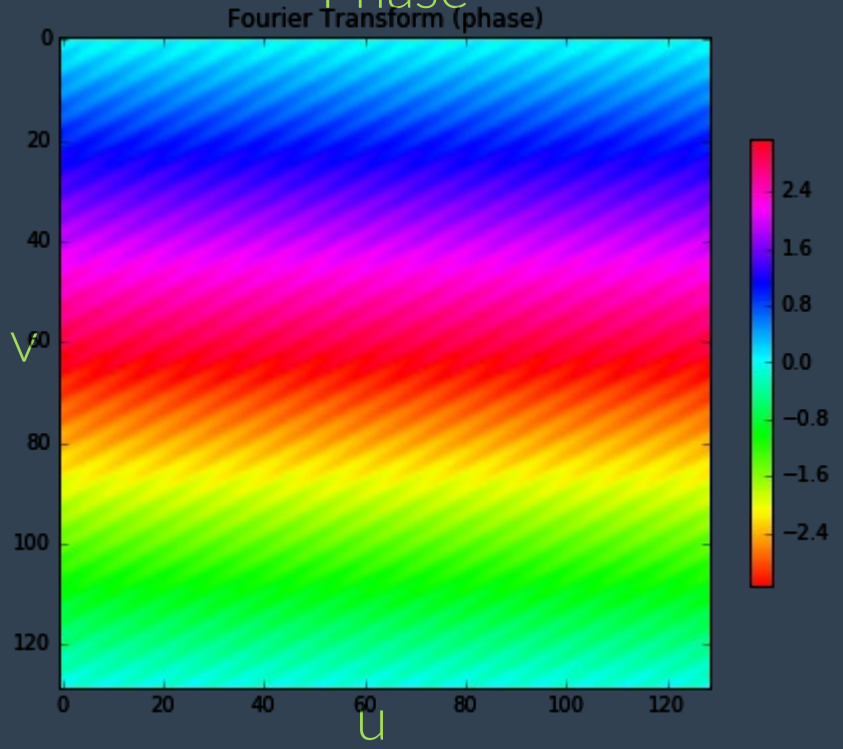


FT(2 Point Sources) [different amplitudes]

Image

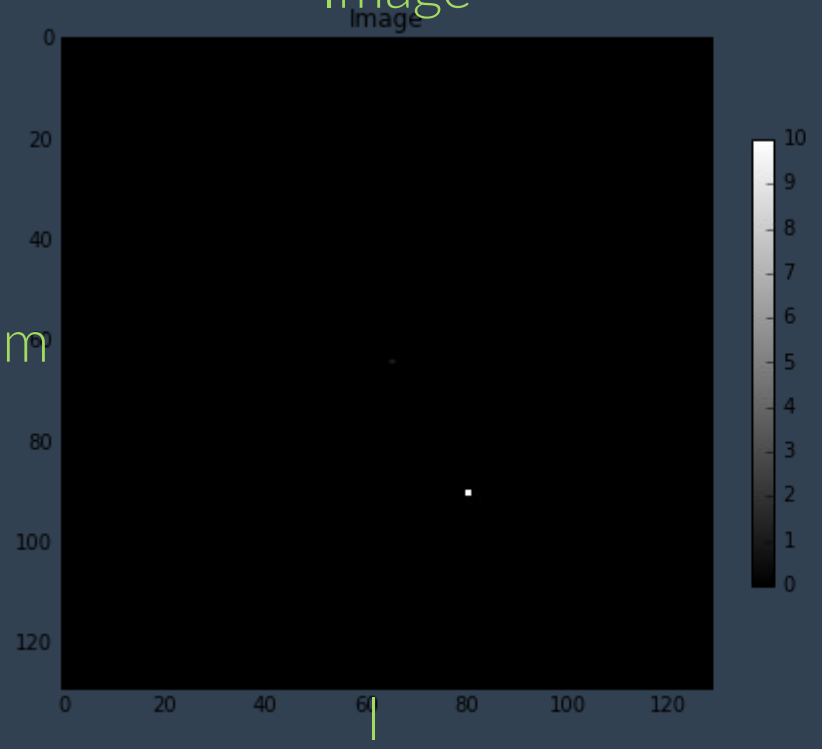


Phase



FT(2 Point Sources) [different amplitudes]

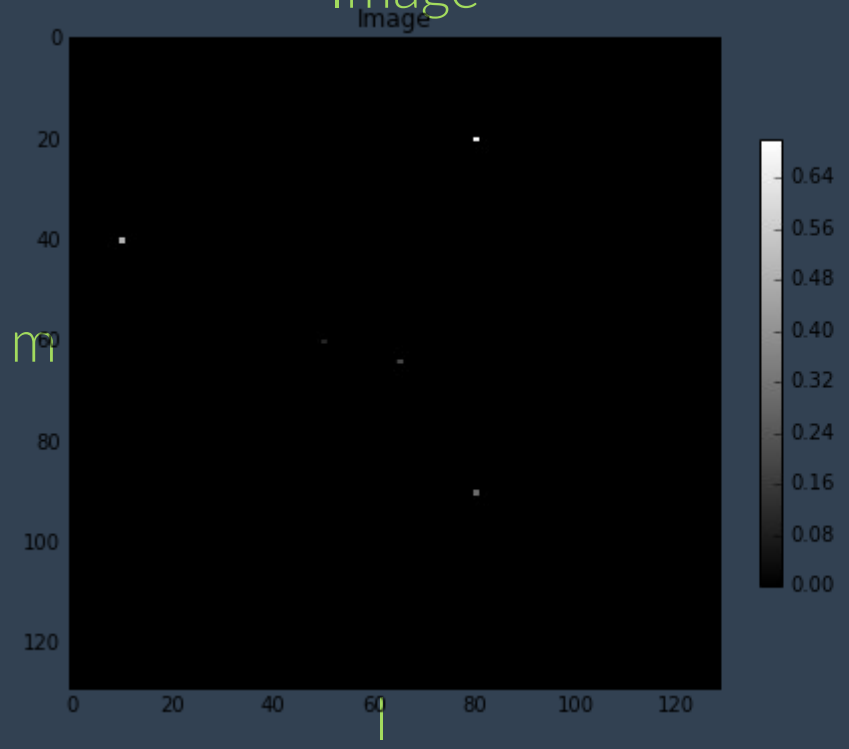
Image



Phase

FT(5 Point Sources) [different amplitudes]

Image



Phase

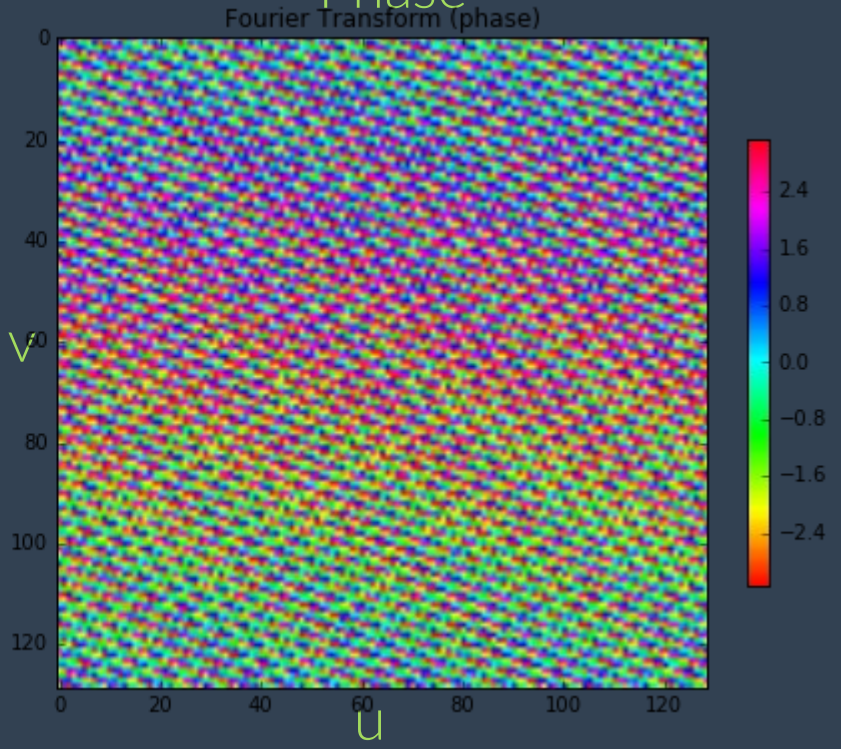
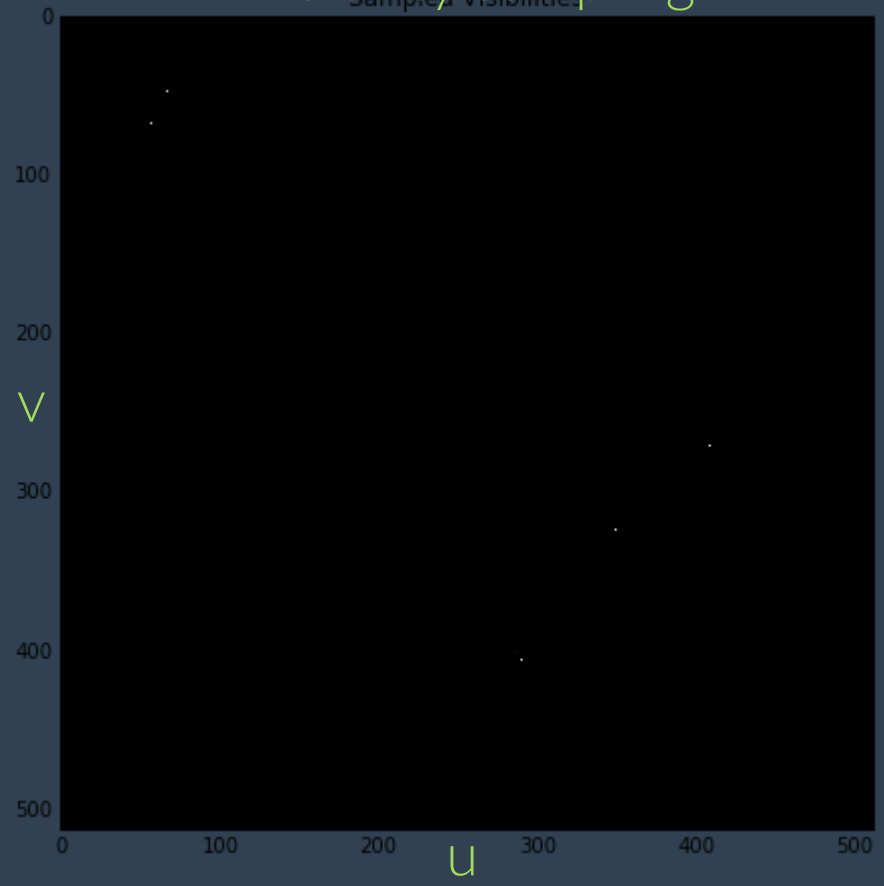
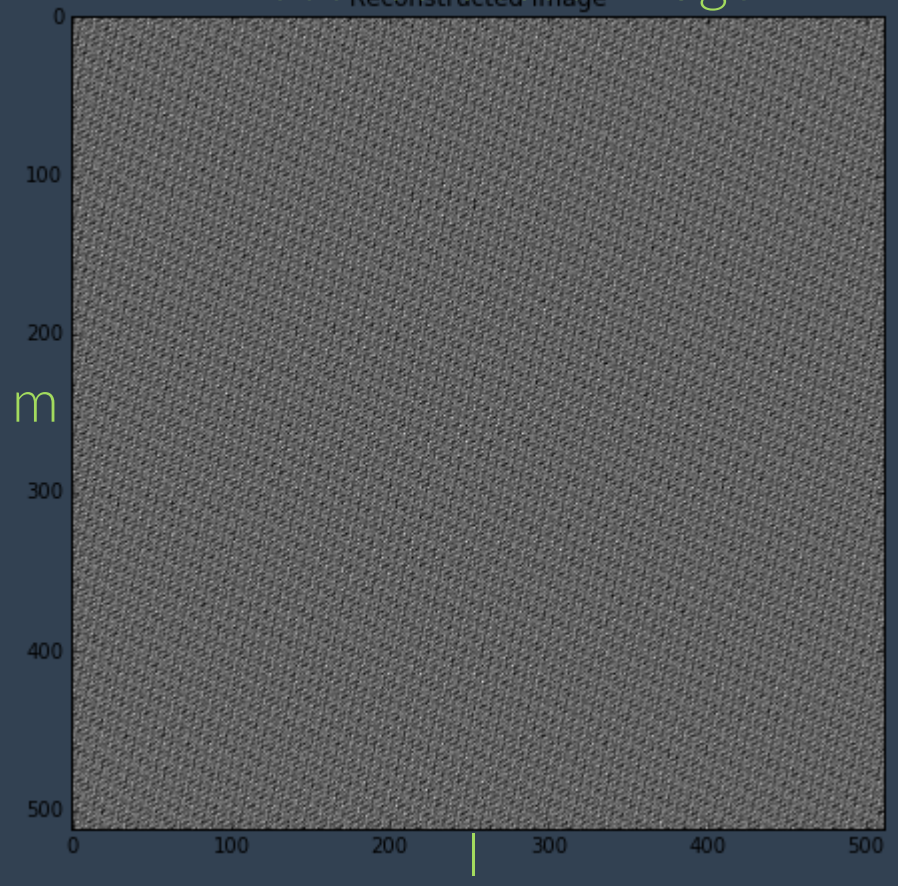


Image Reconstruction from Sampling Visibilities

Visibility Sampling



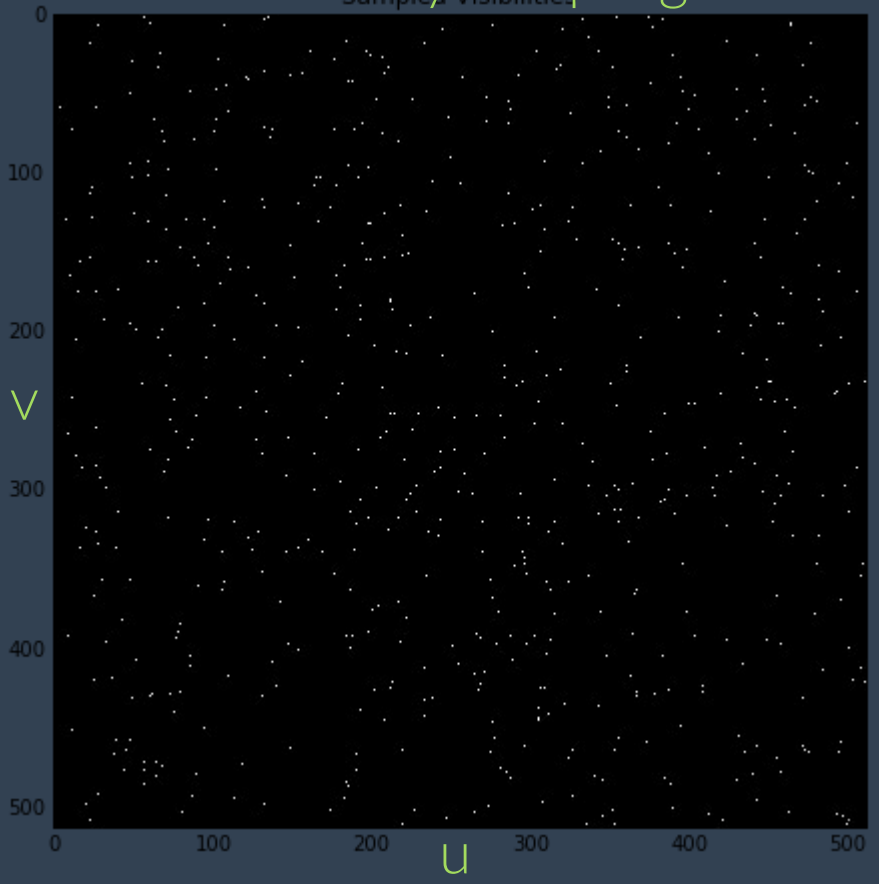
Reconstructed Image



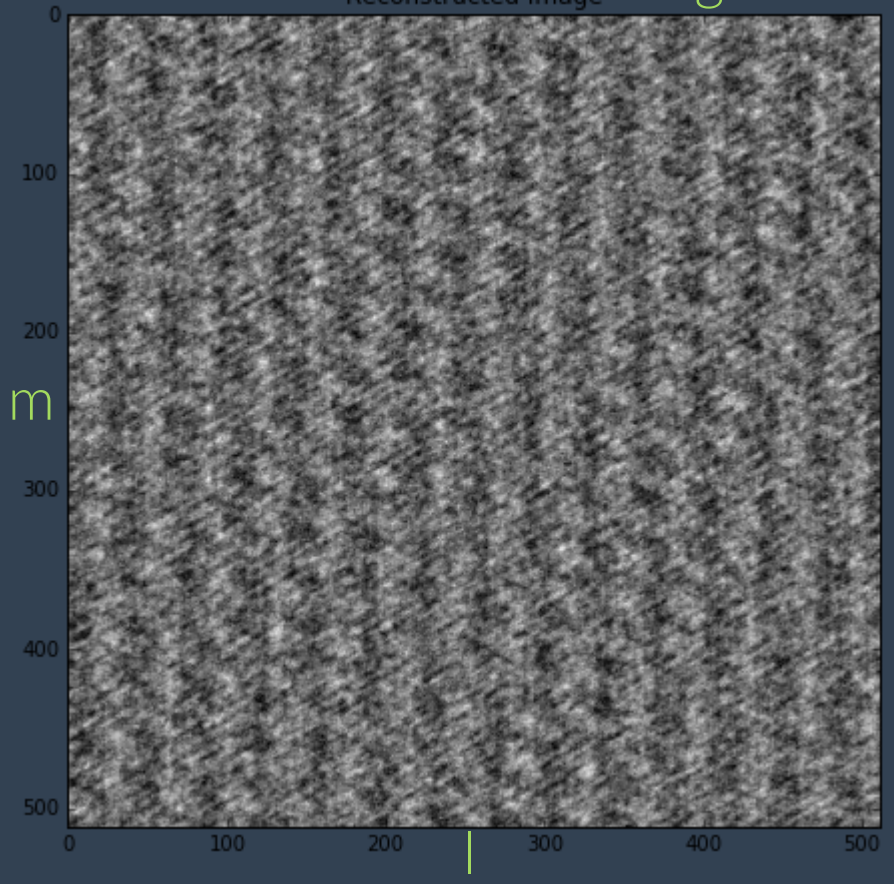
$N_{sampling} = 10$

Image Reconstruction from Sampling Visibilities

Visibility Sampling



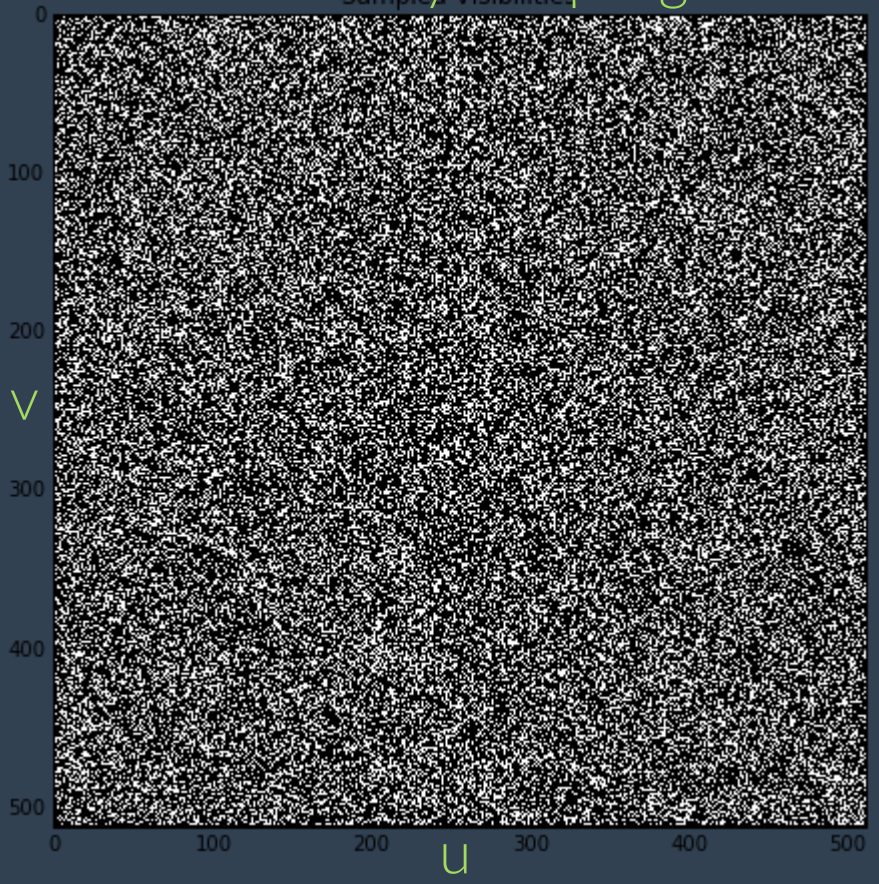
Reconstructed Image



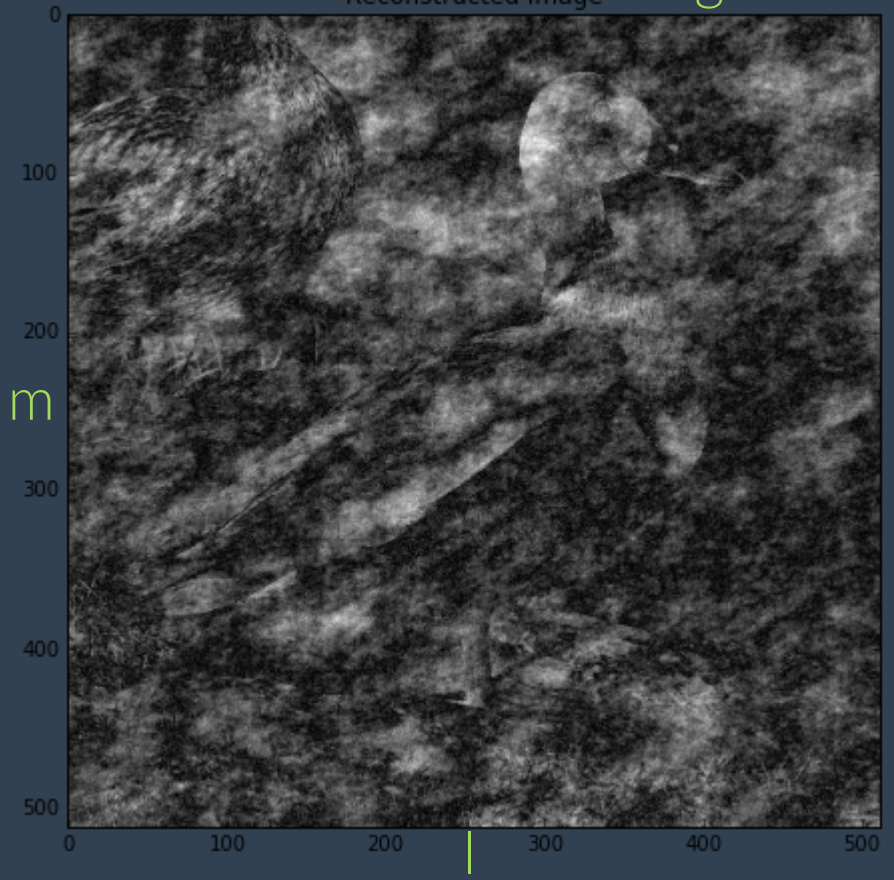
$N_{sampling} = 1{,}000$

Image Reconstruction from Sampling Visibilities

Visibility Sampling



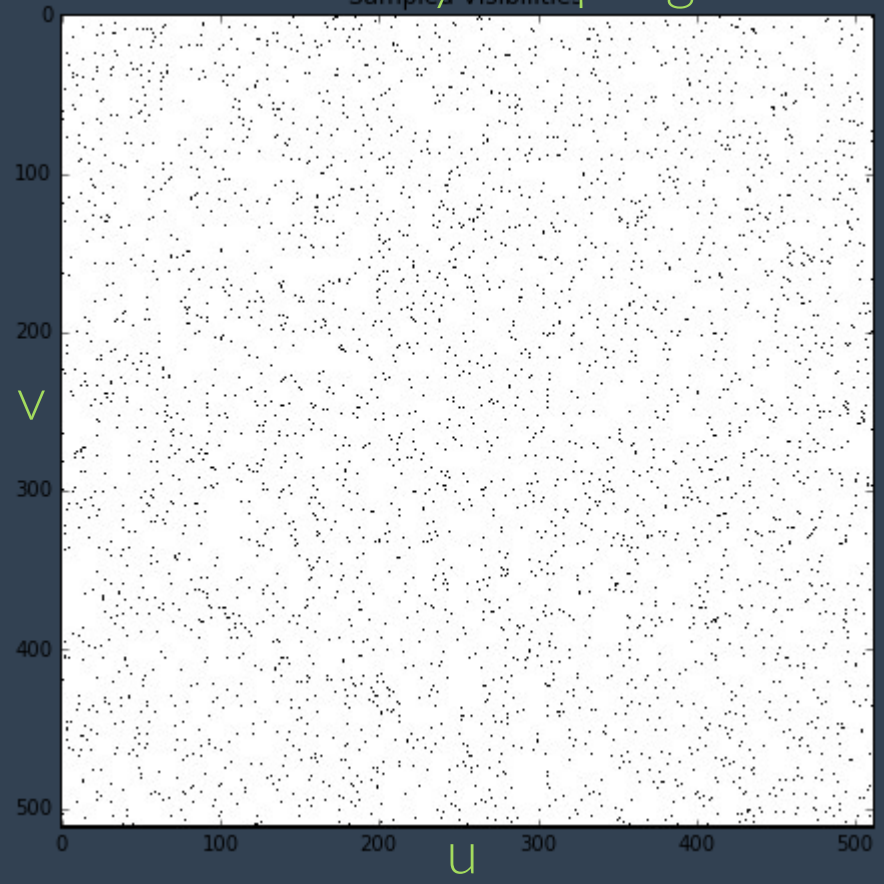
Reconstructed Image



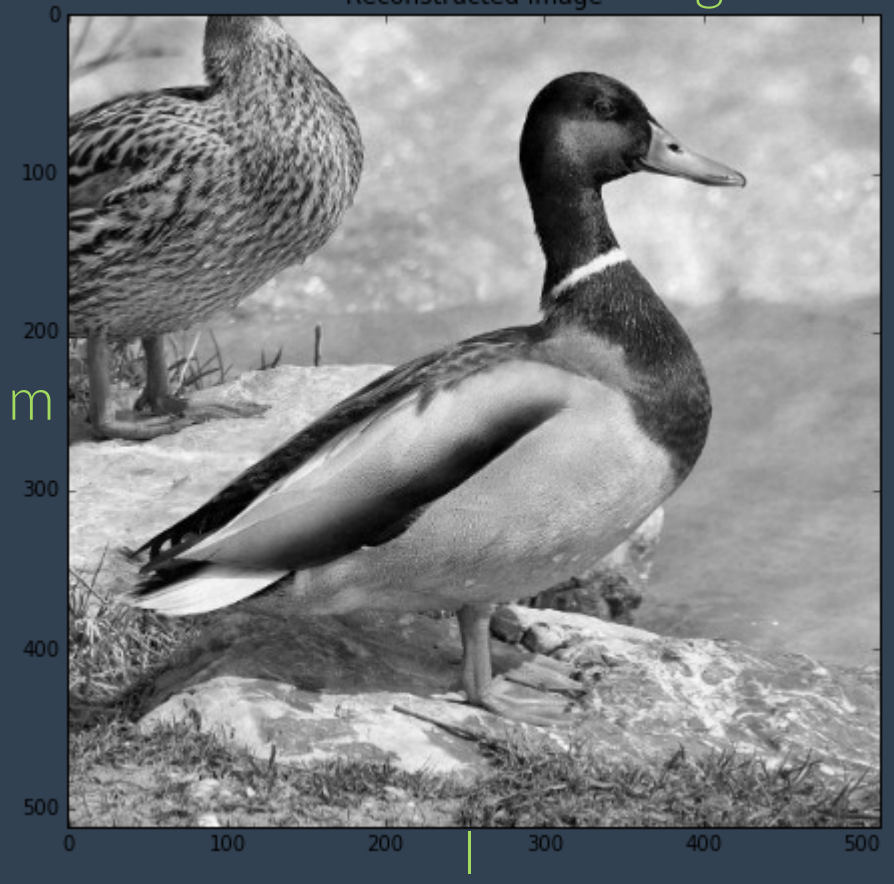
$N_{sampling} = 100{,}000$

Image Reconstruction from Sampling Visibilities

Visibility Sampling



Reconstructed Image



$N_{sampling} = 1{,}000{,}000$


Effects of the Point
Spread Function

Sampling and Point Spread Functions

We do not fully sample the visibility plane, due to engineering and physical limitations we only sample based upon some sampling function

$$S(u,v) \rightleftharpoons \mathrm{PSF}(l,m)$$

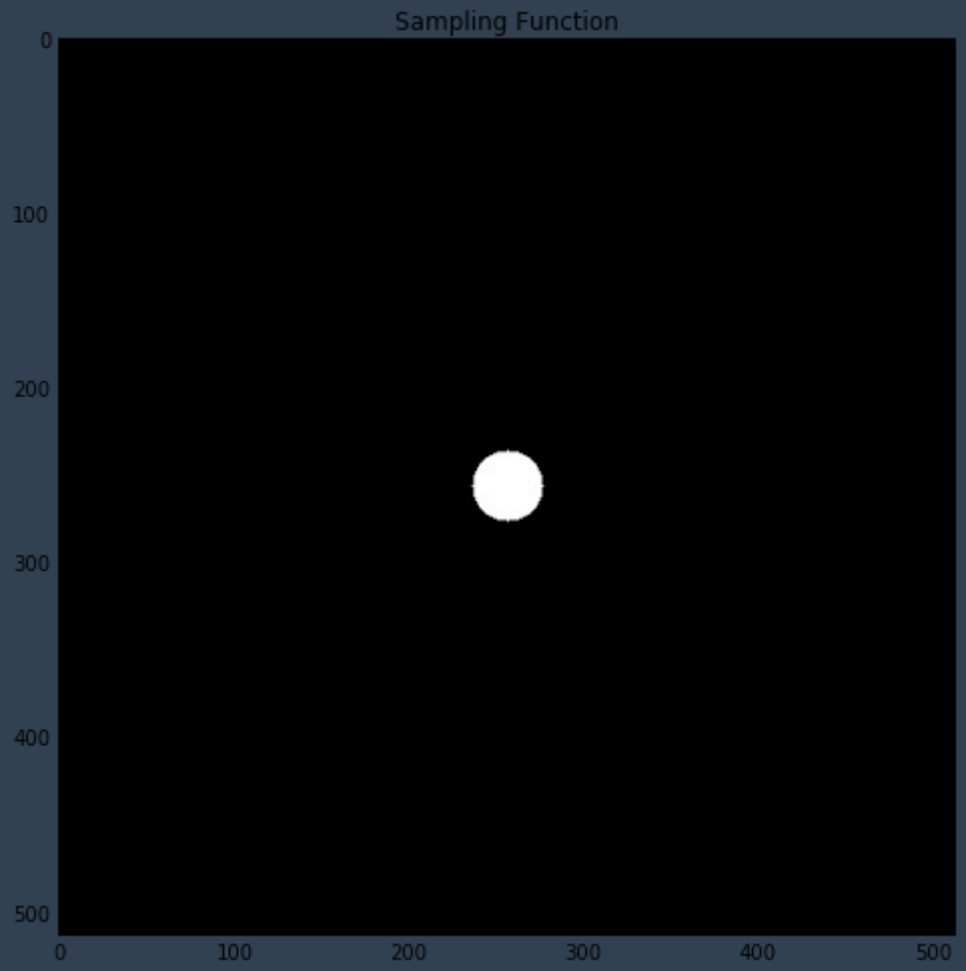
The Point Spread Function (PSF) is a spatial filter due to the limited sampling, it is the Fourier Transform of the sampling function.

Sampling and Point Spread Functions

Due to the limited sampling of the visibility plane the observed image of the sky is the convolution of the true sky with the array Point Spread Function

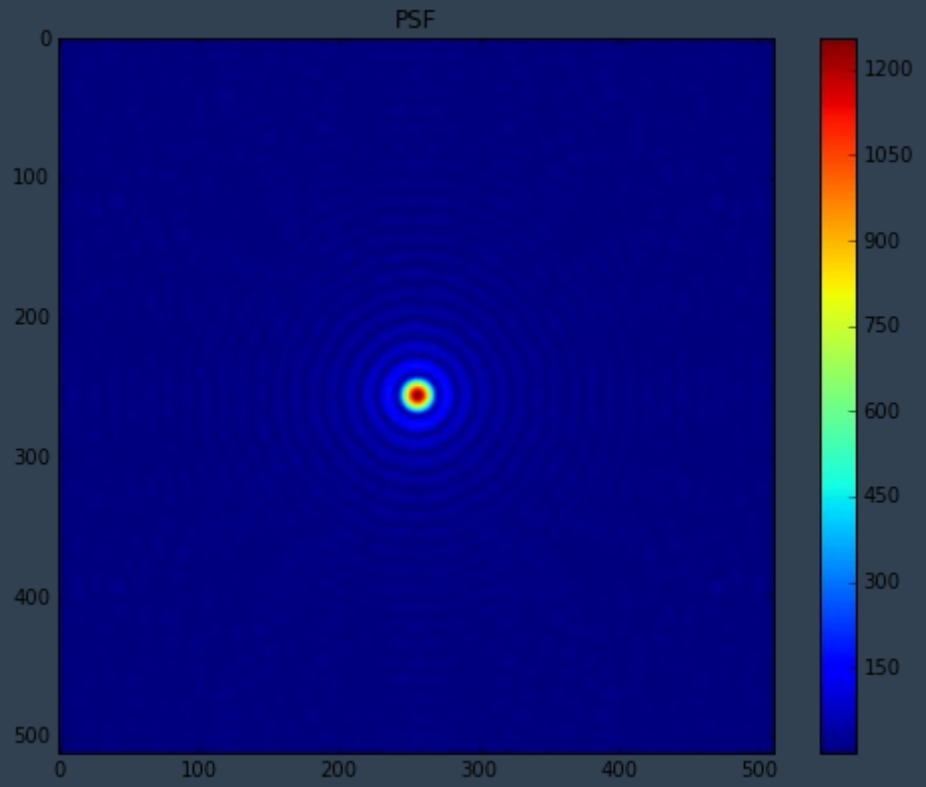
$$I_{\mathrm{obs}} = \mathrm{PSF} \circ I_{\mathrm{true}}$$

Visibility Sampling Function



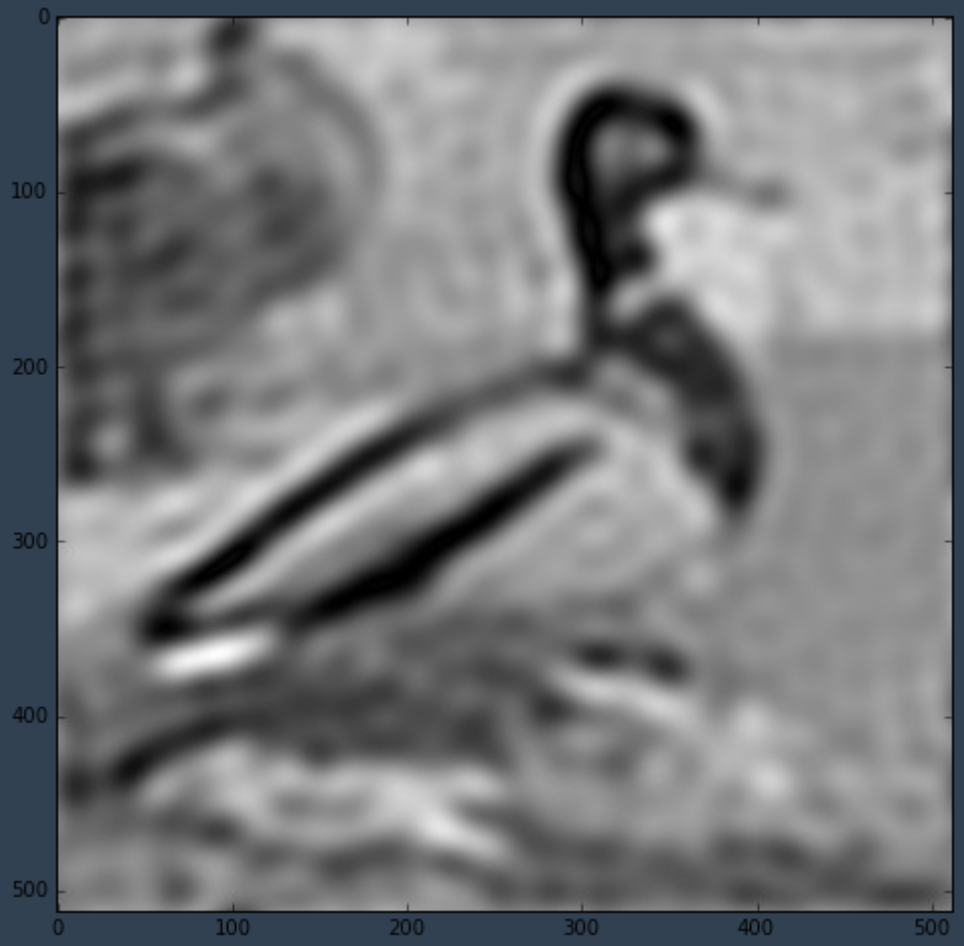
$$S(u,v)$$

Point Spread Function



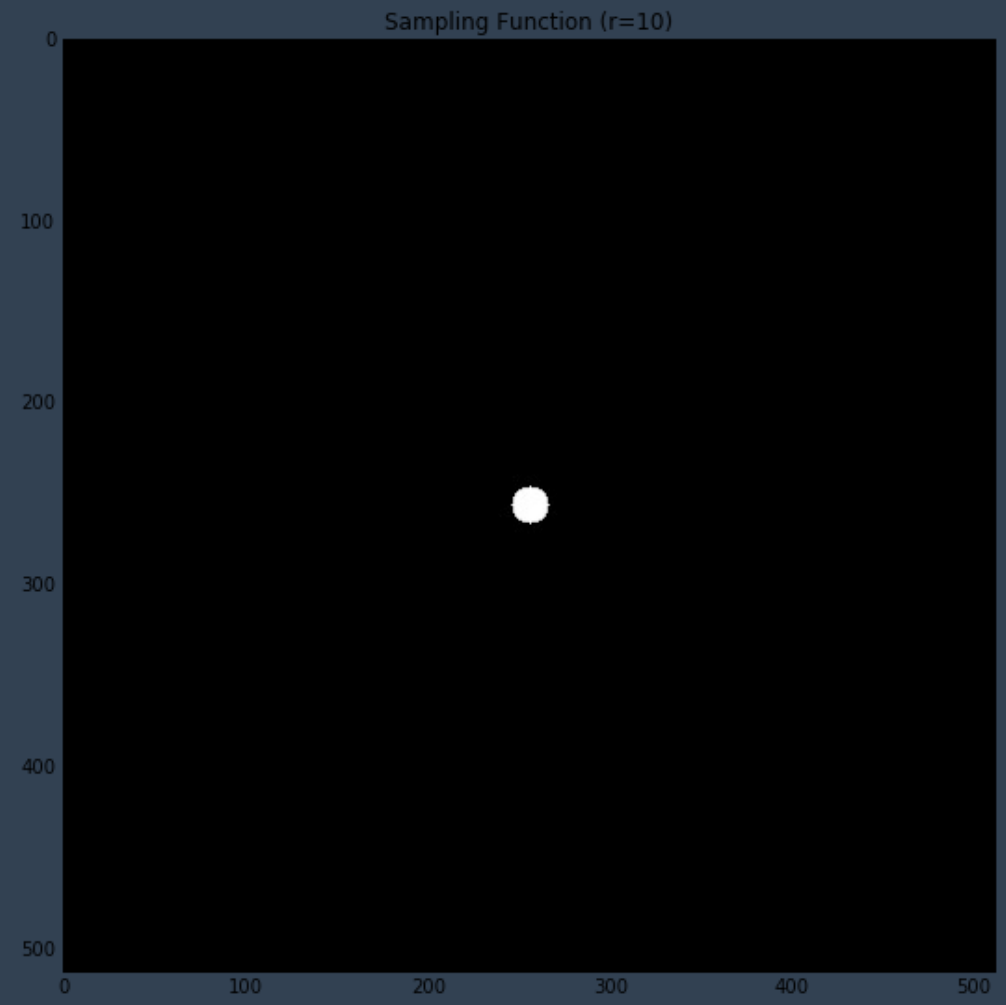
$$\mathrm{PSF}(l, m)$$

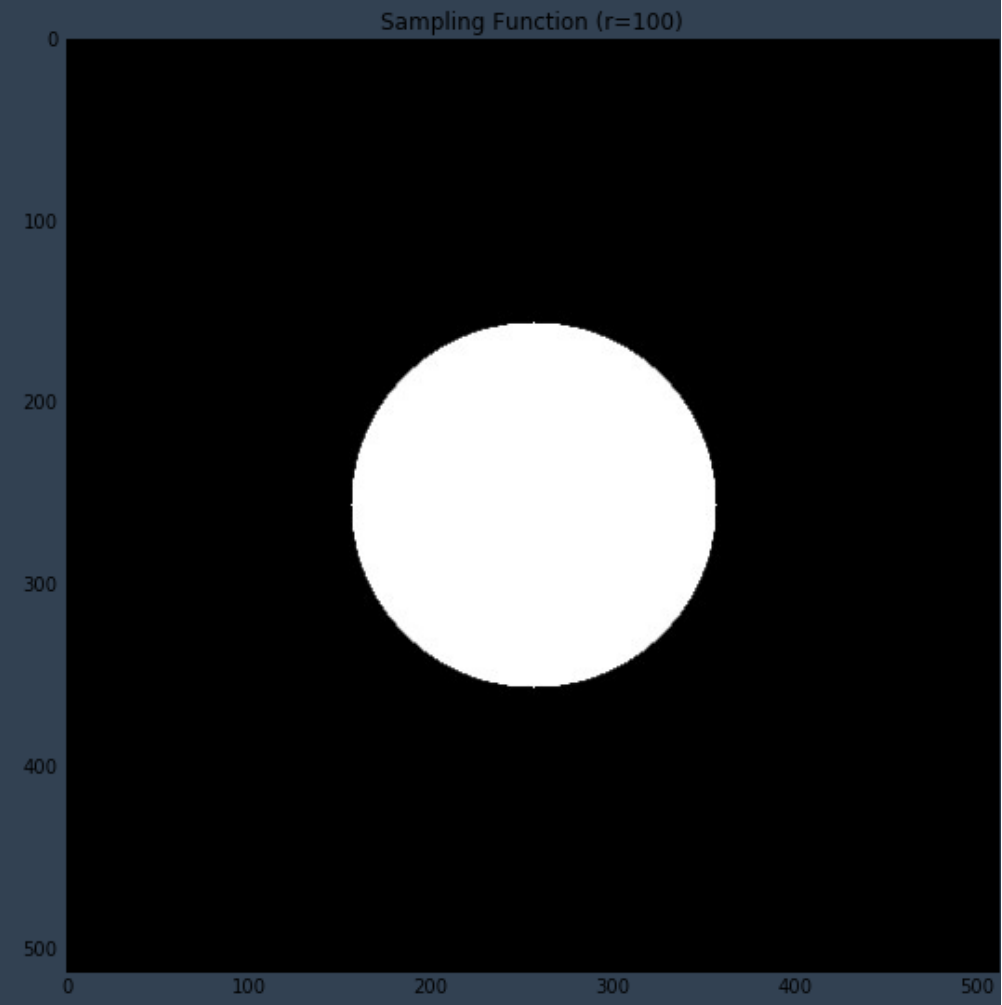
PSF Filtered Image



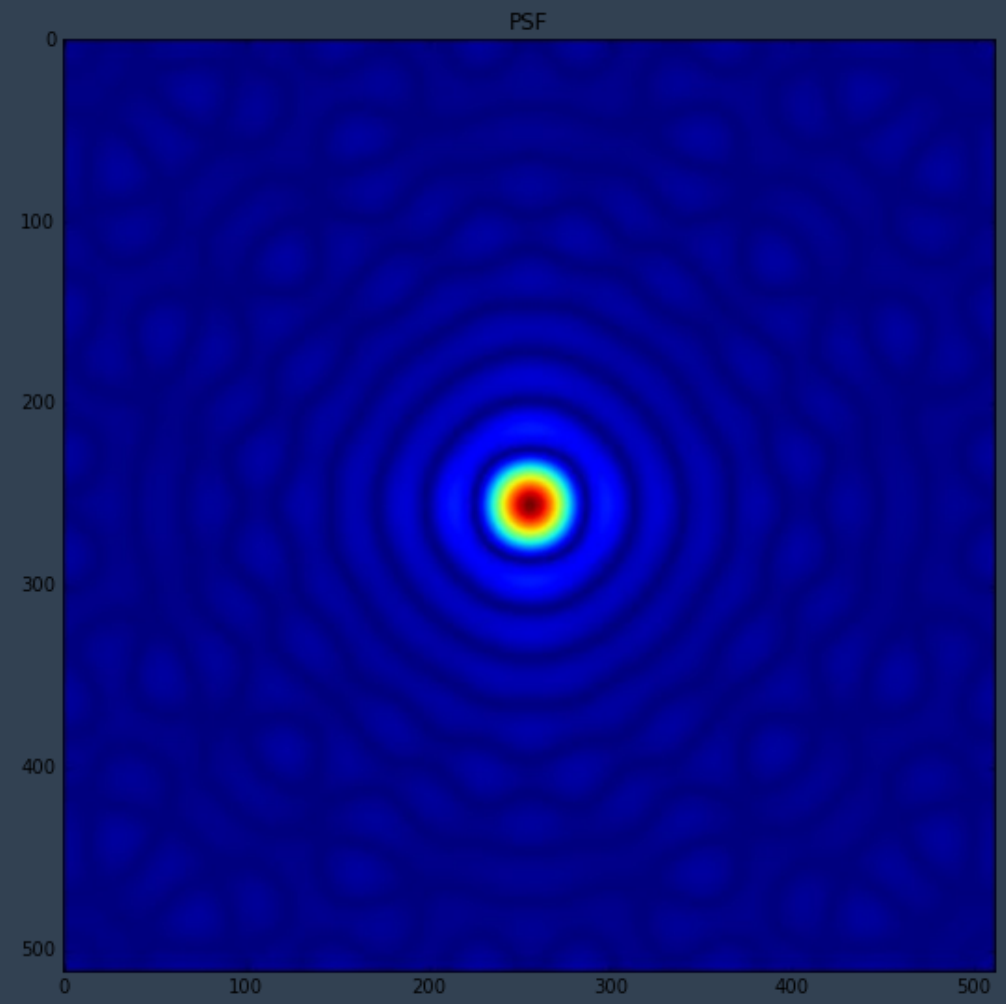
$$I_{\text{obs}} = \text{PSF} \circ I_{\text{true}}$$

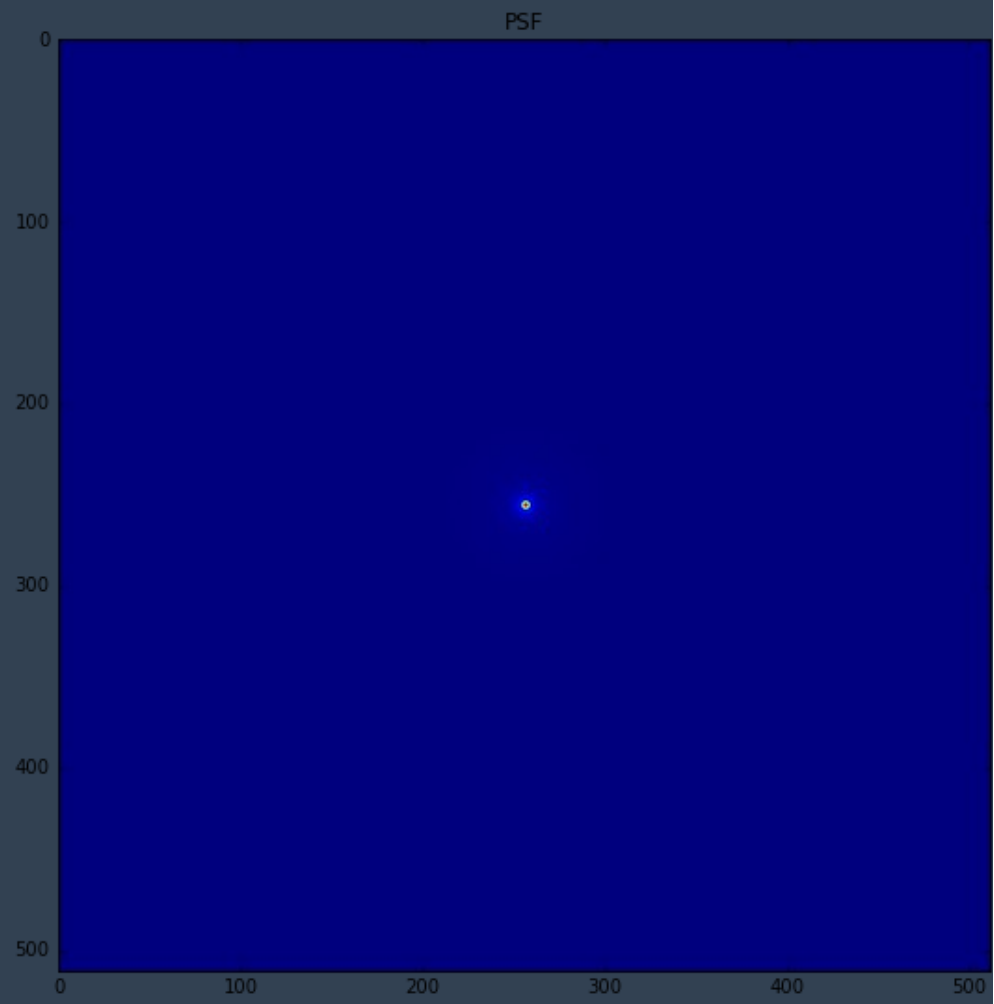
Visibility Sampling Functions



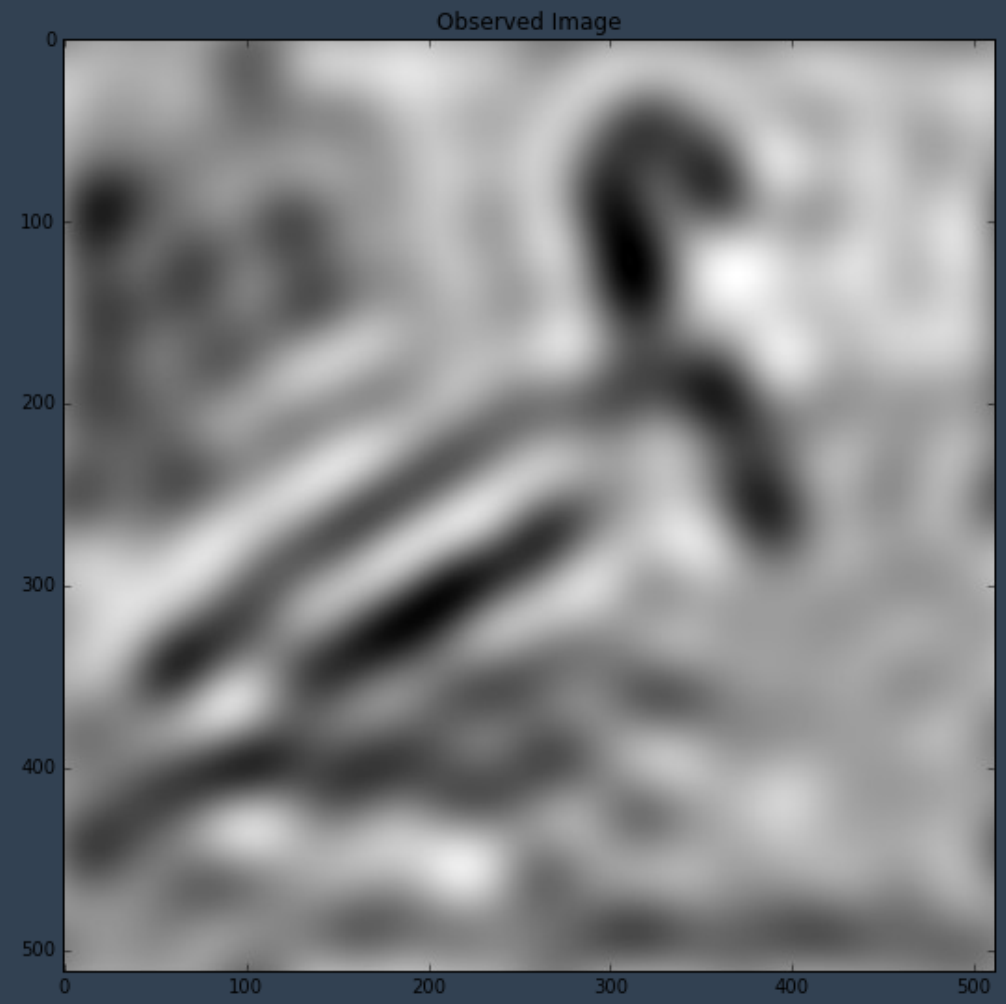


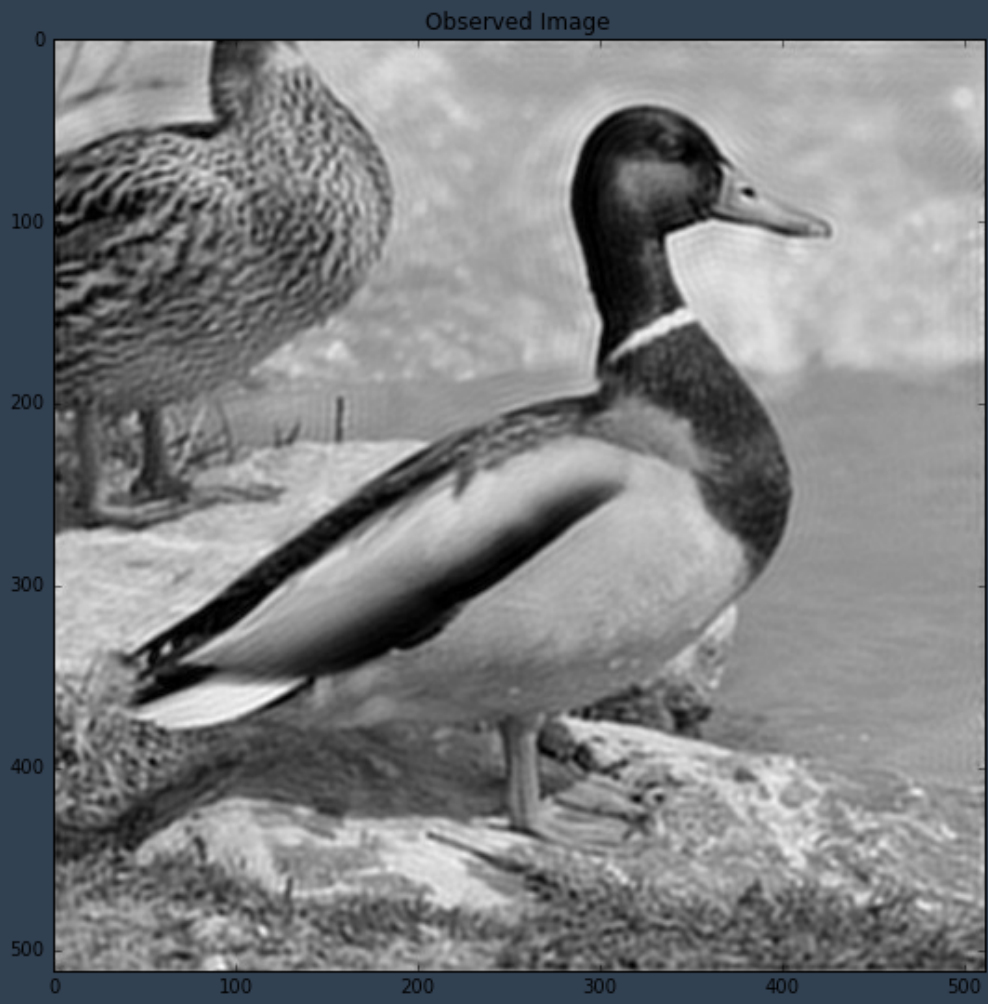
Point Spread Functions

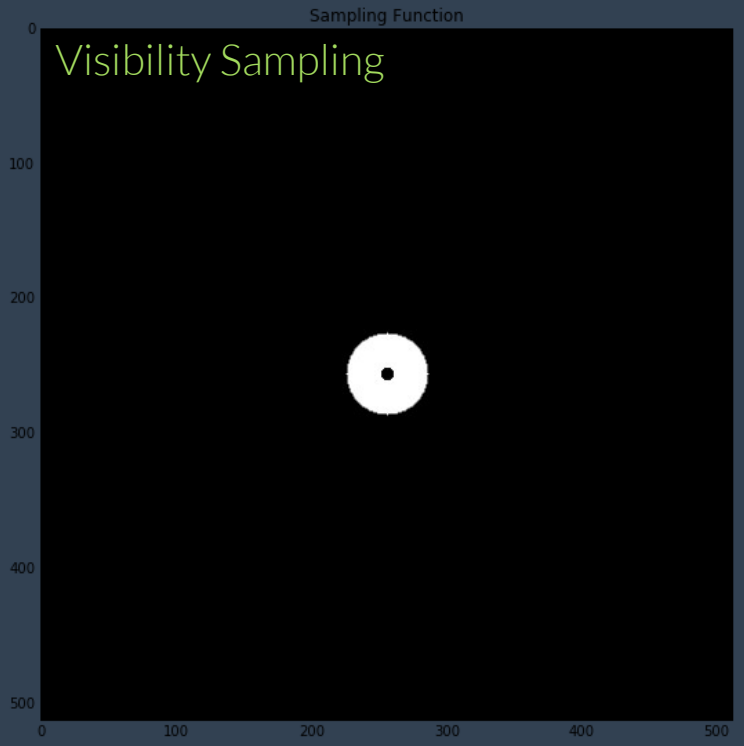


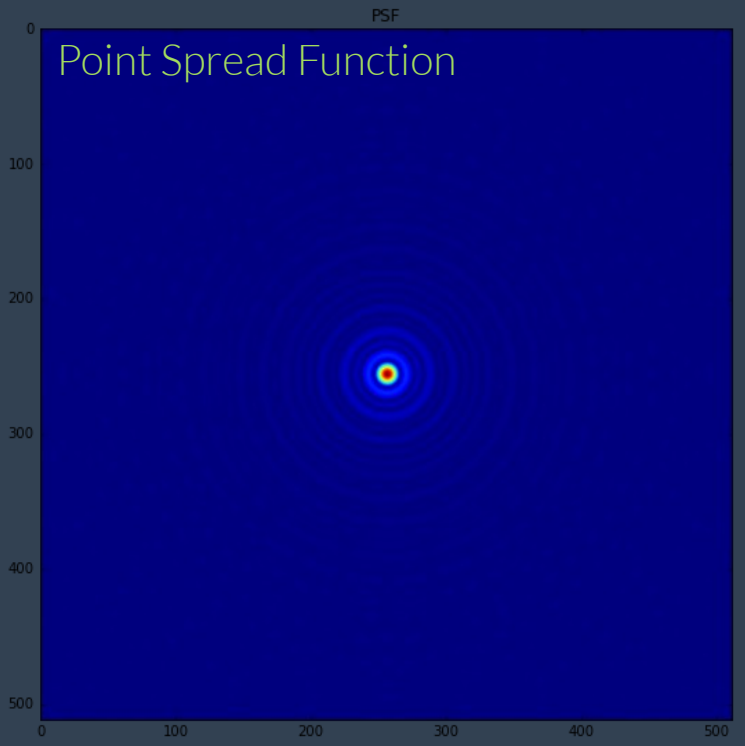


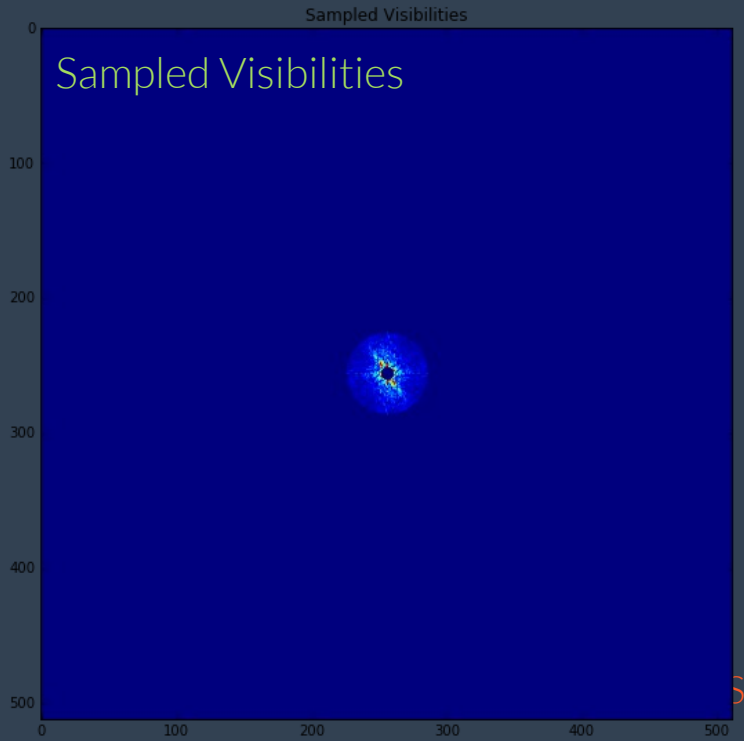
PSF Filtered Images



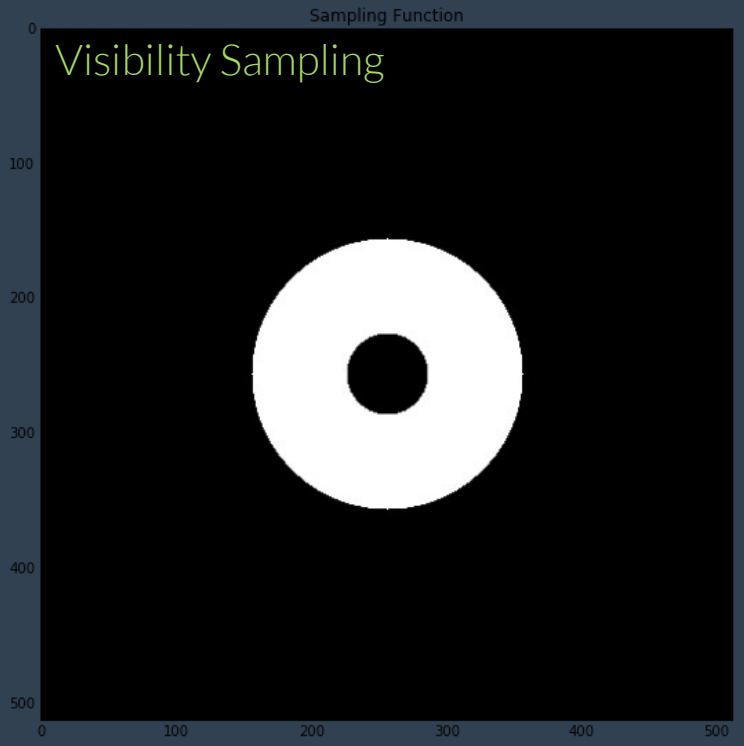


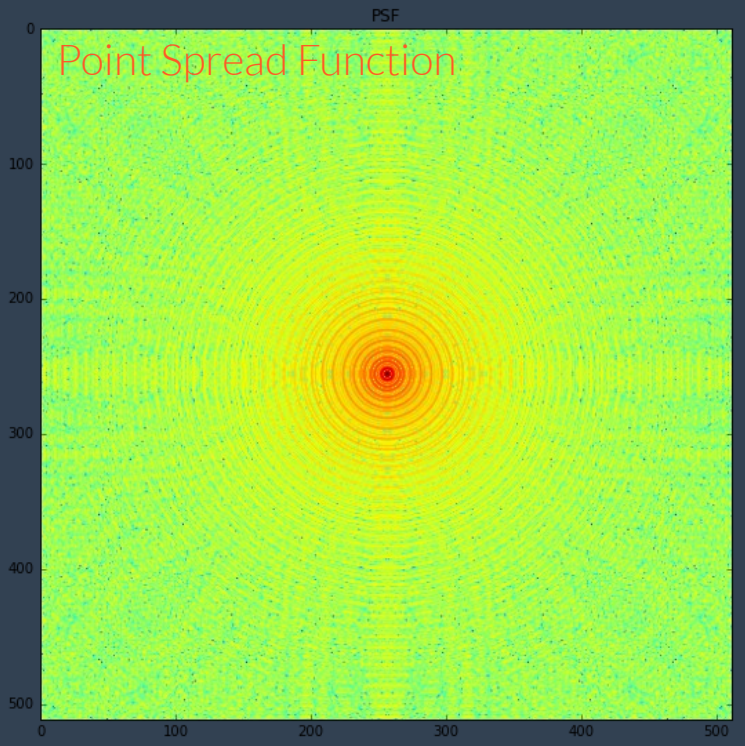

Sampling Function
Visibility Sampling
0
100
200
300
400
500

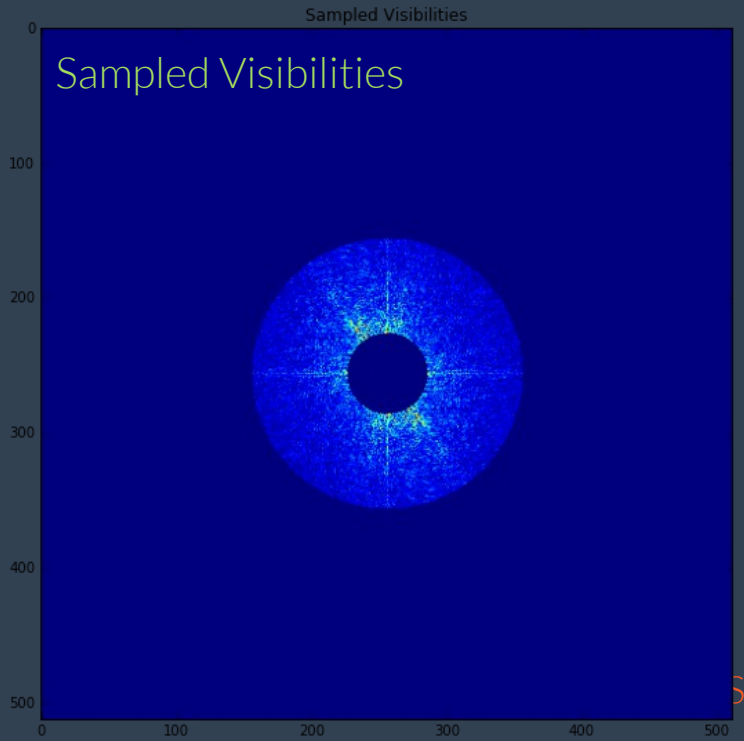

PSF
Point Spread Function
0
100
200
300
400
500

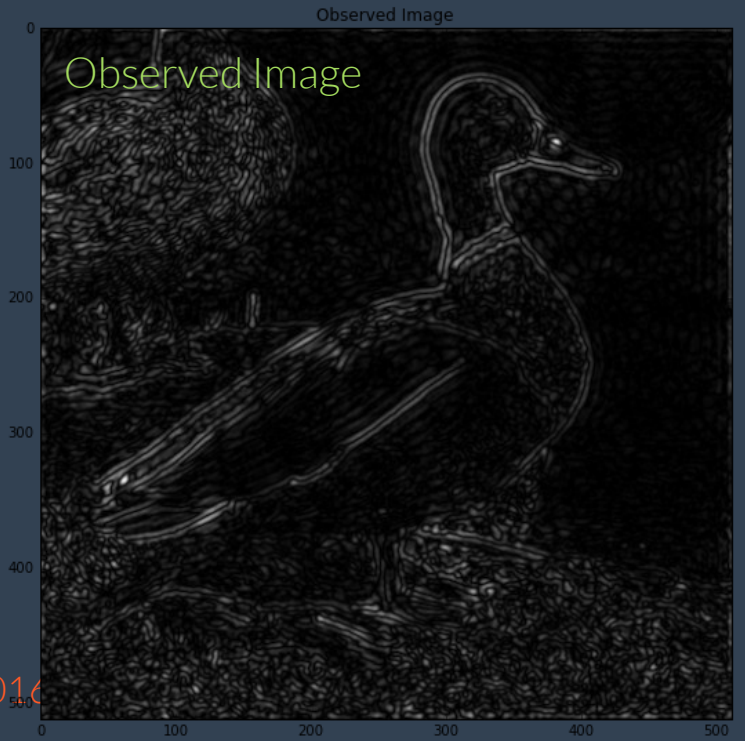

Sampled Visibilities
Sampled Visibilities
0
100
200
300
400
500

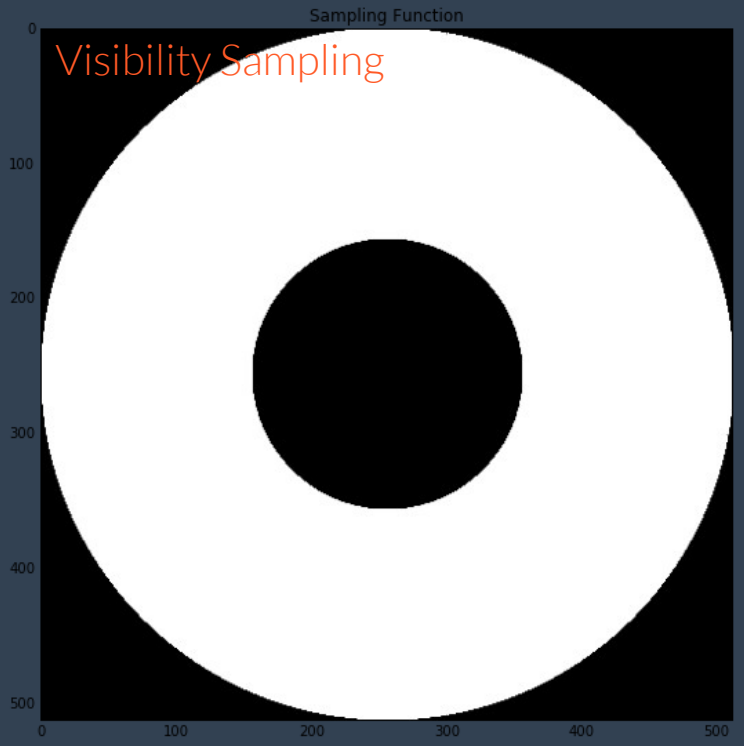

Observed Image
Observed Image
0
100
200
300
400
500

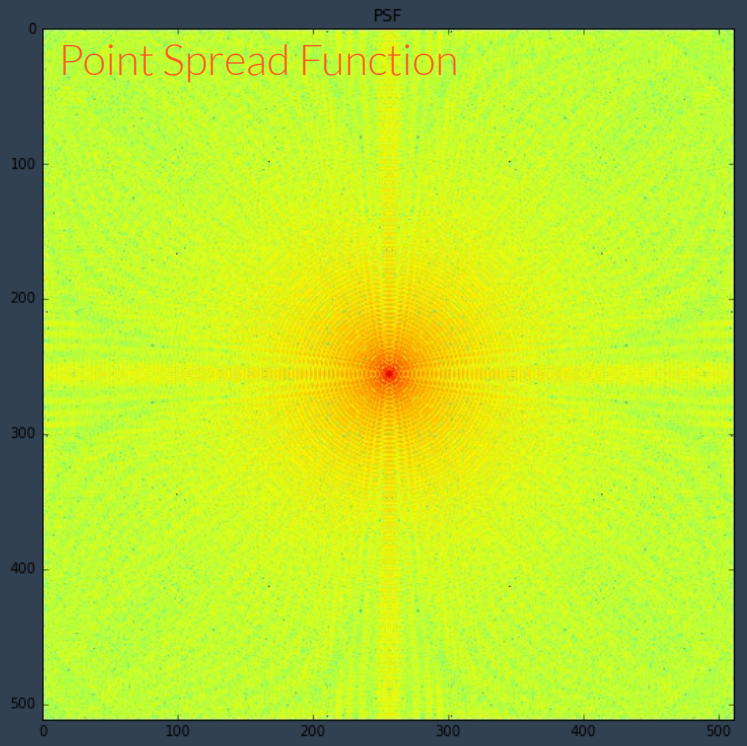

Sampling Function
Visibility Sampling

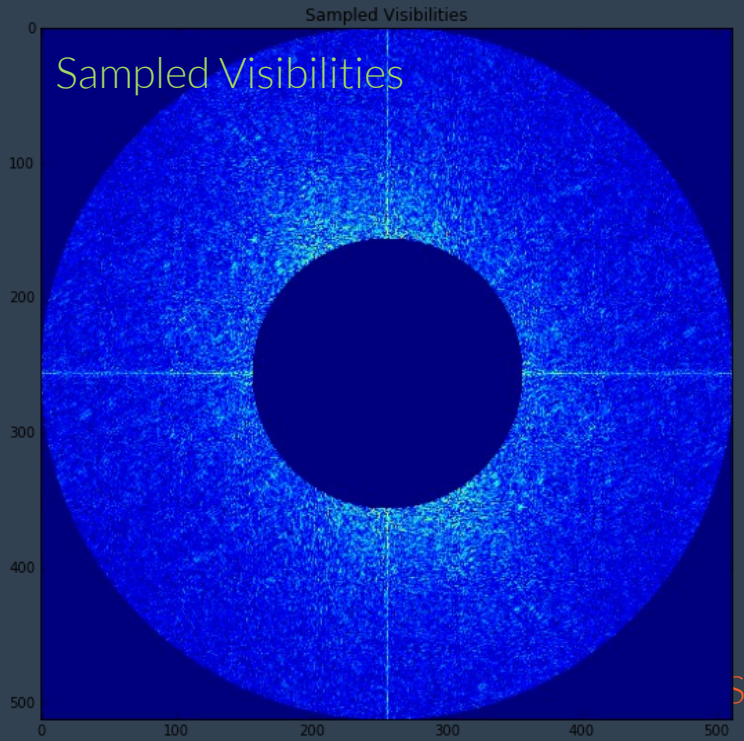

PSF
Point Spread Function


Sampled Visibilities
Sampled Visibilities


Observed Image
Observed Image

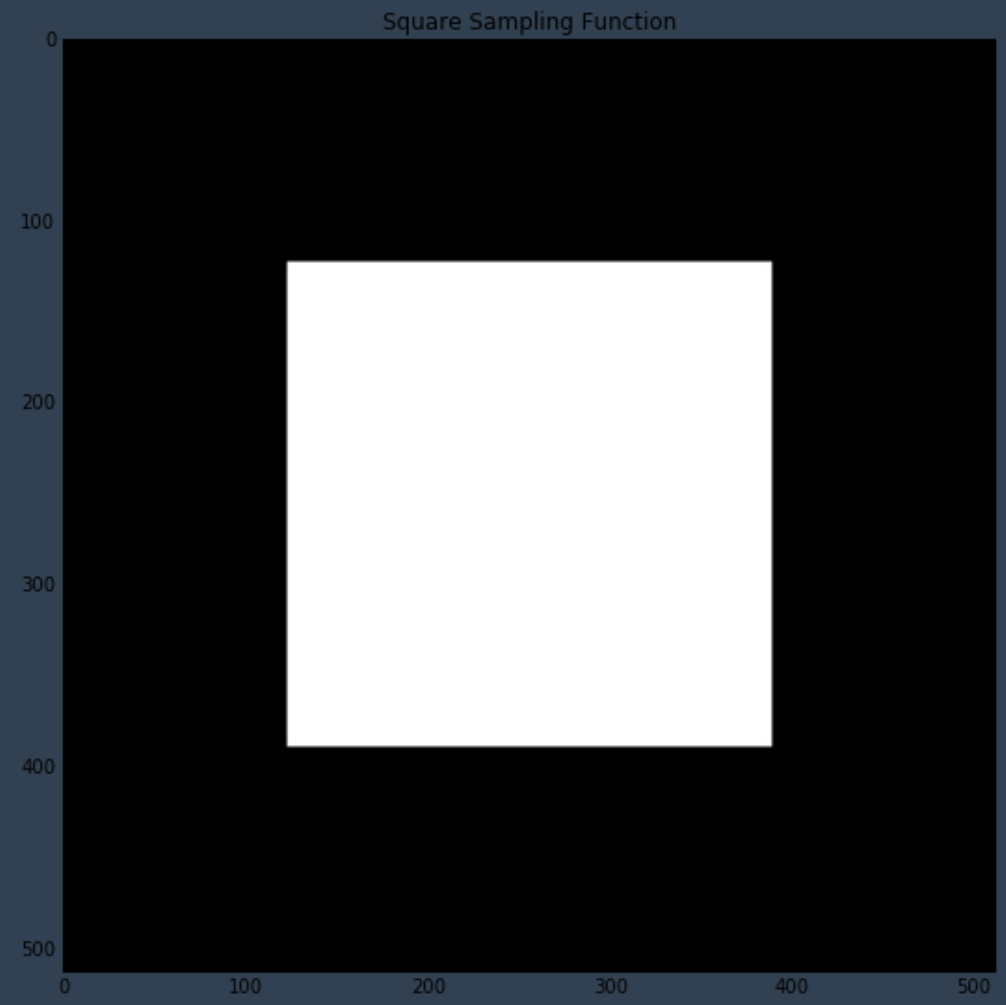

Sampling Function
Visibility Sampling

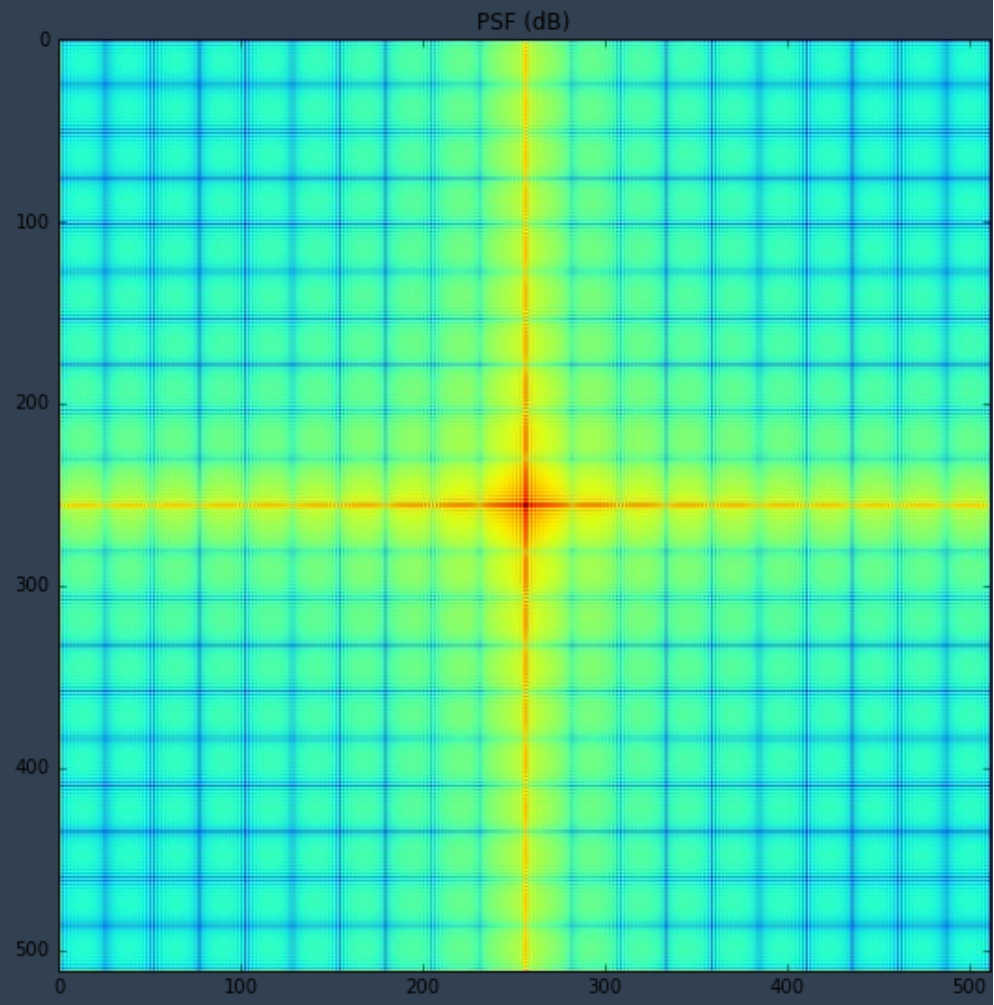

PSF
Point Spread Function


Sampled Visibilities
Sampled Visibilities

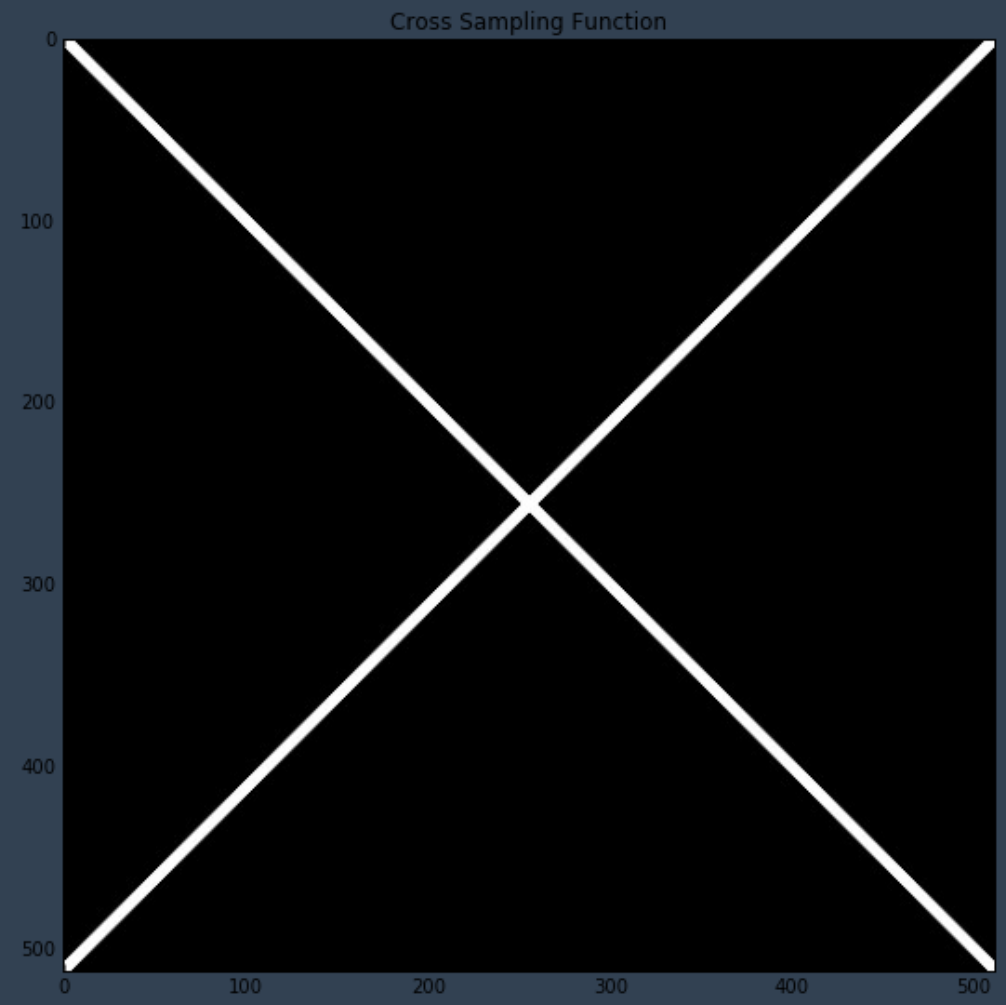

Observed Image
Observed Image

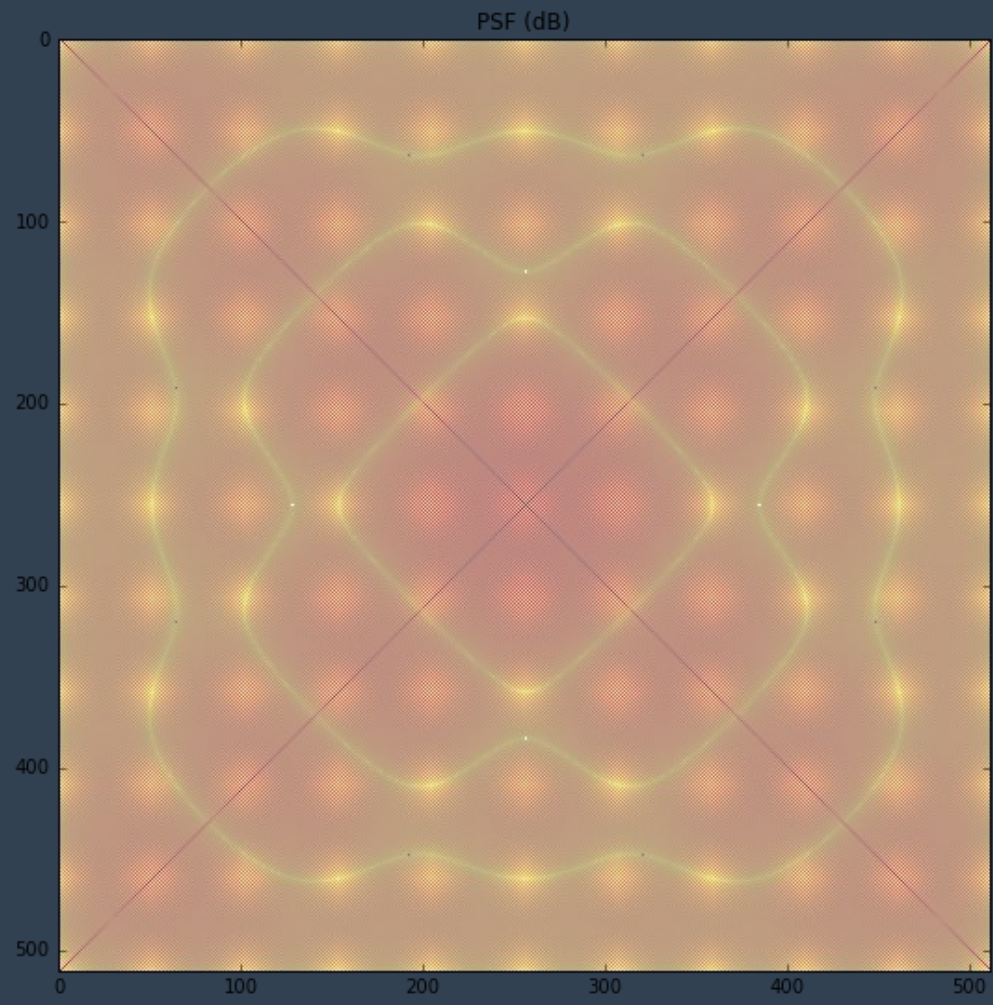
Square Sampling Function



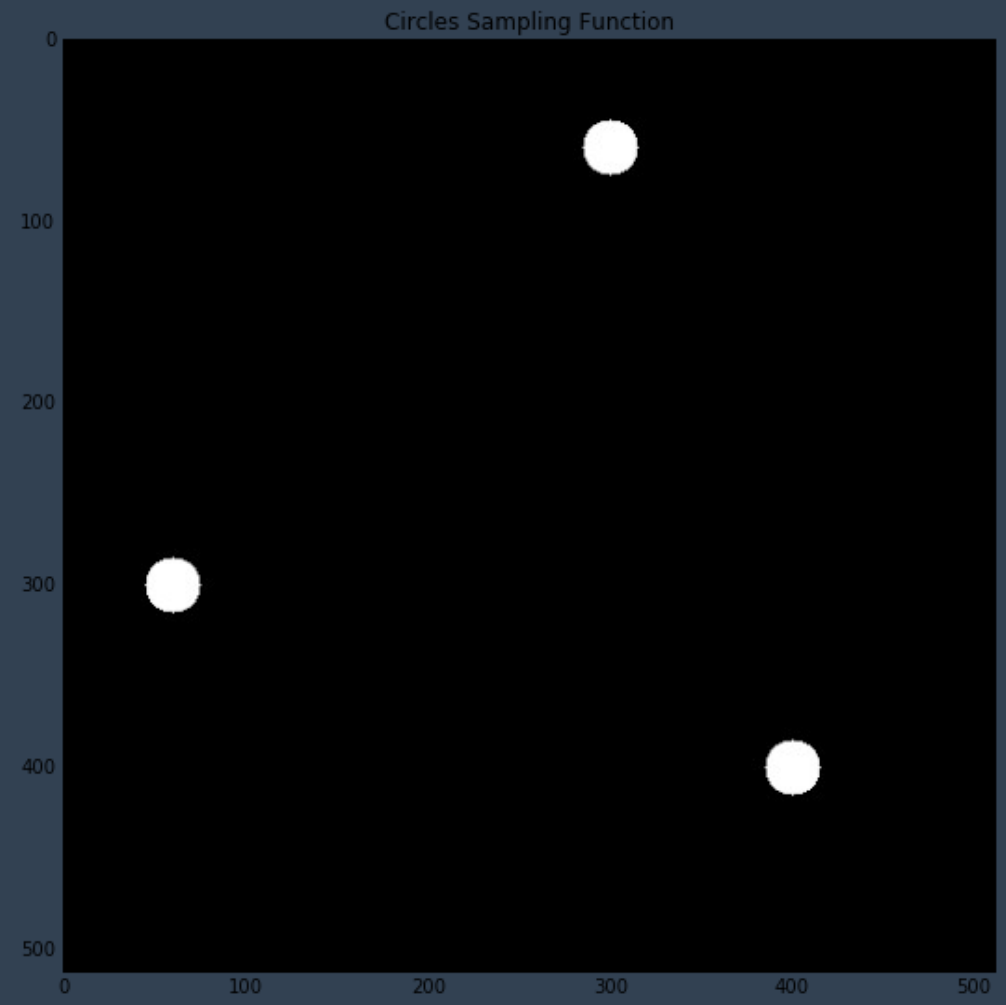


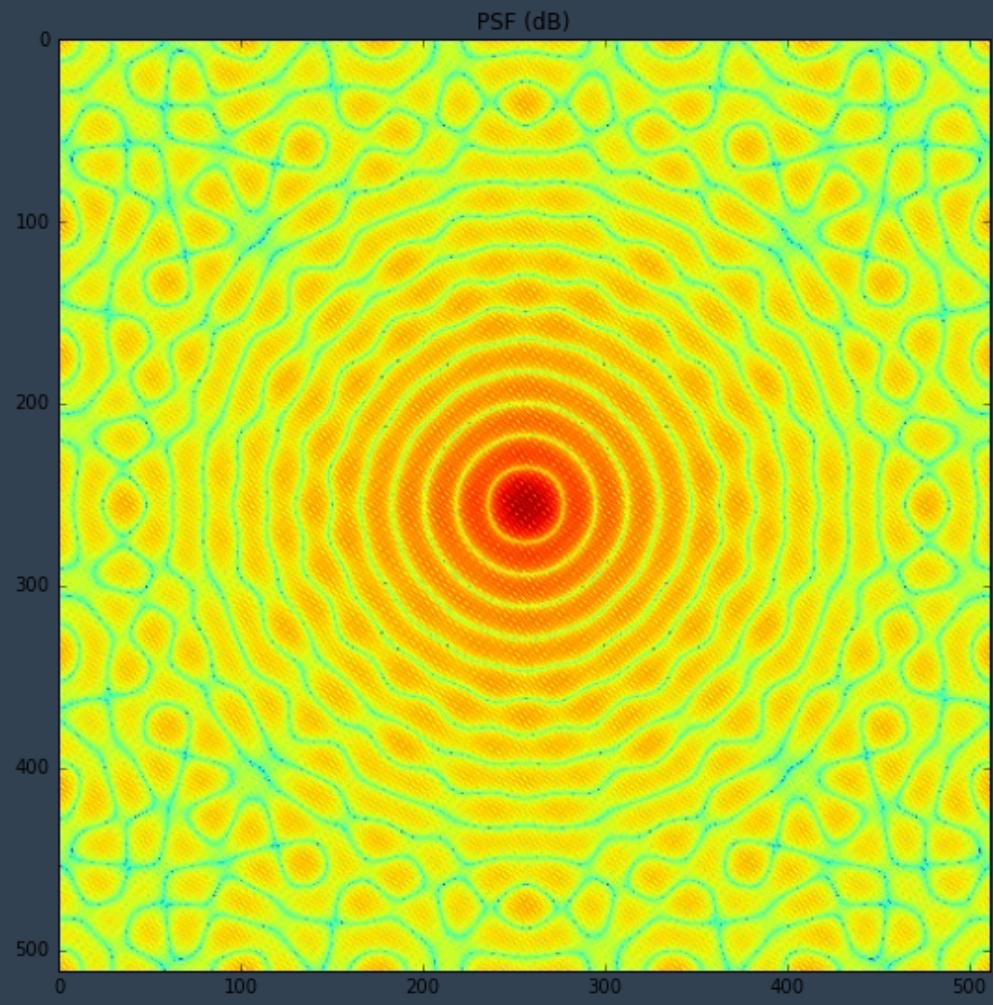
Cross Sampling Function



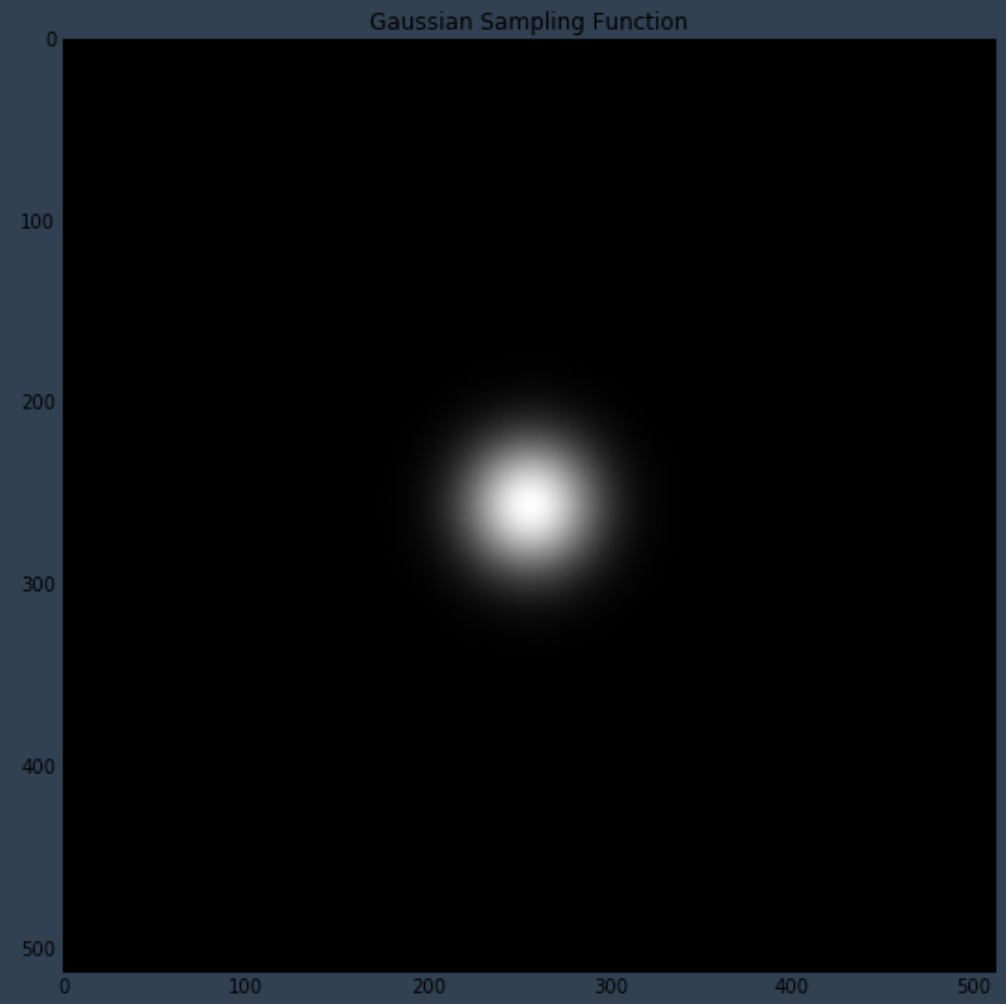


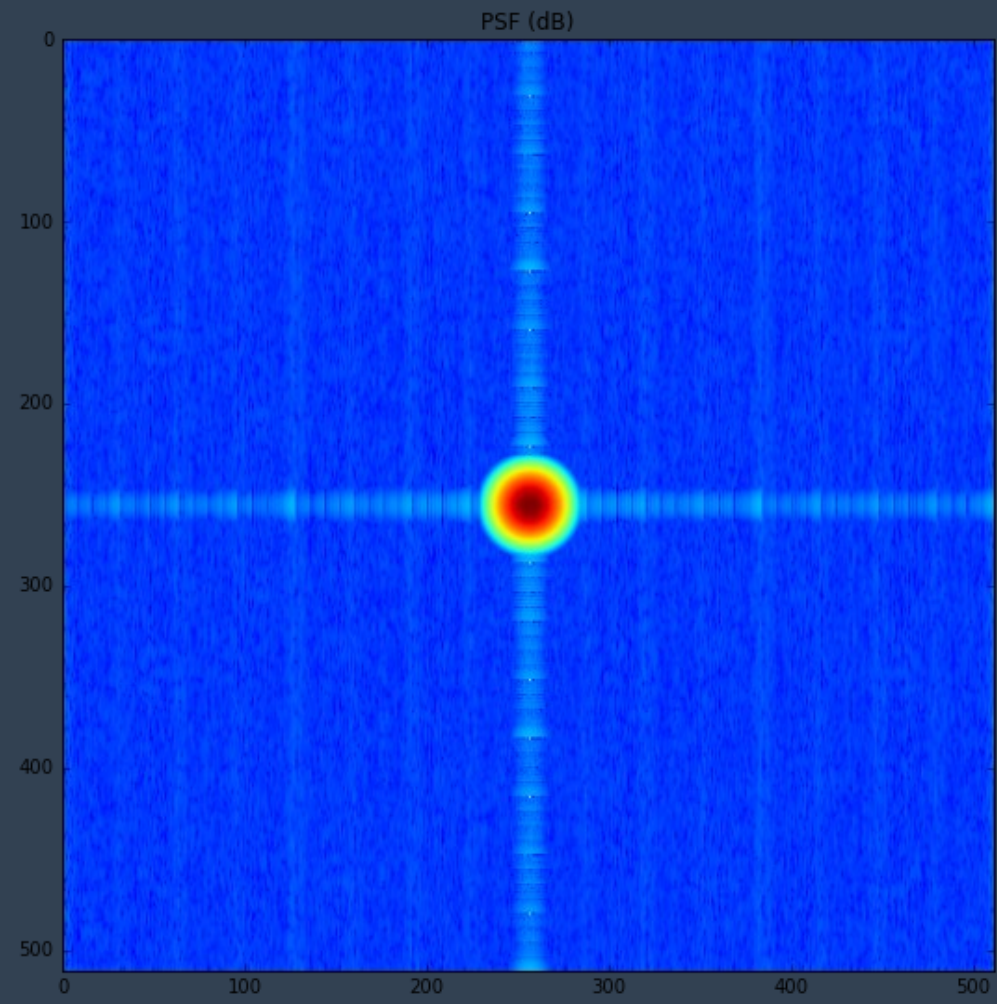
Small Circles Sampling Function



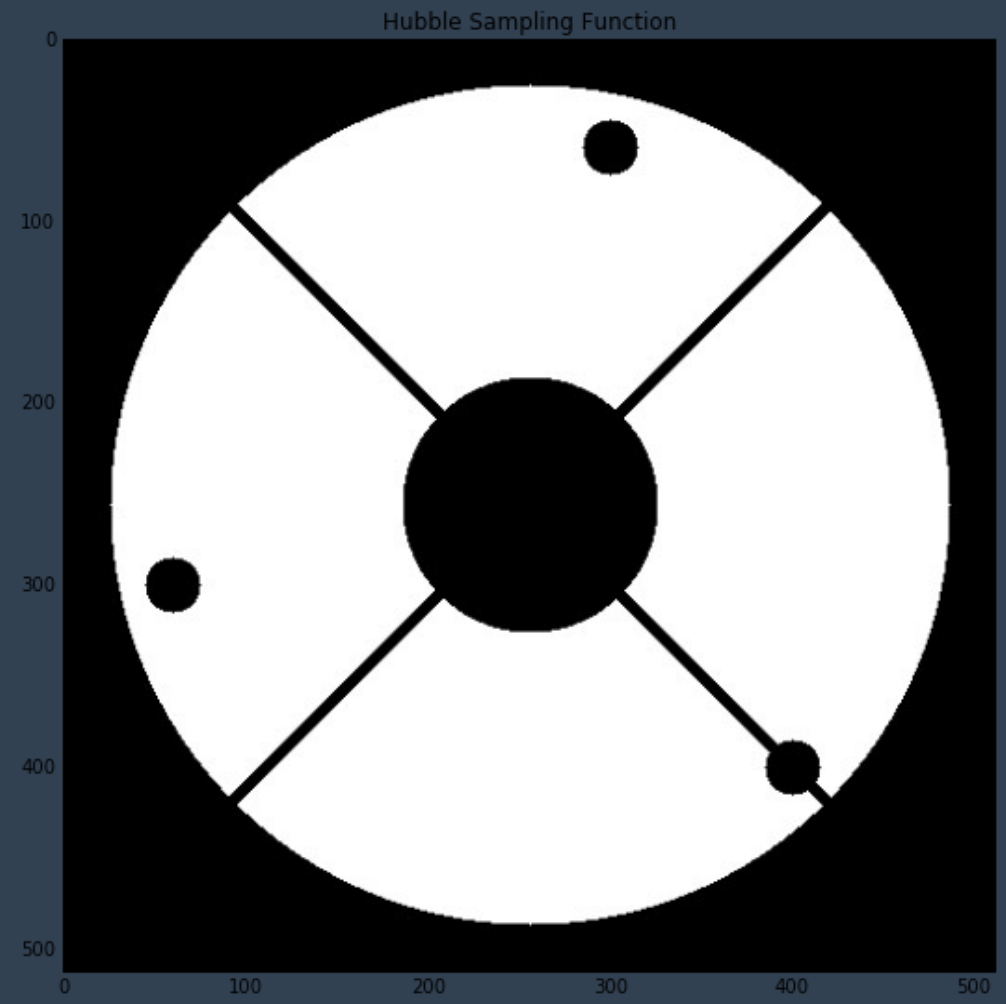


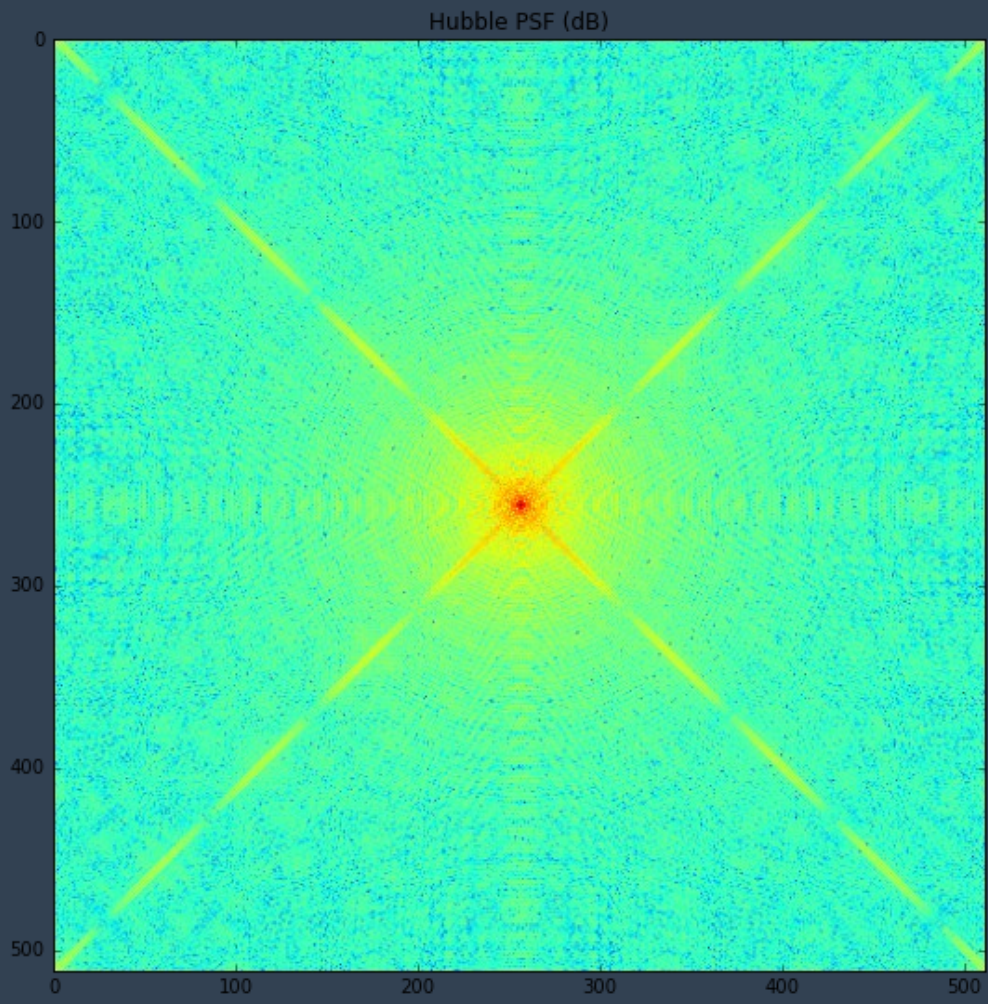
Gaussian Sampling Function





Hubble Telescope Sampling Function

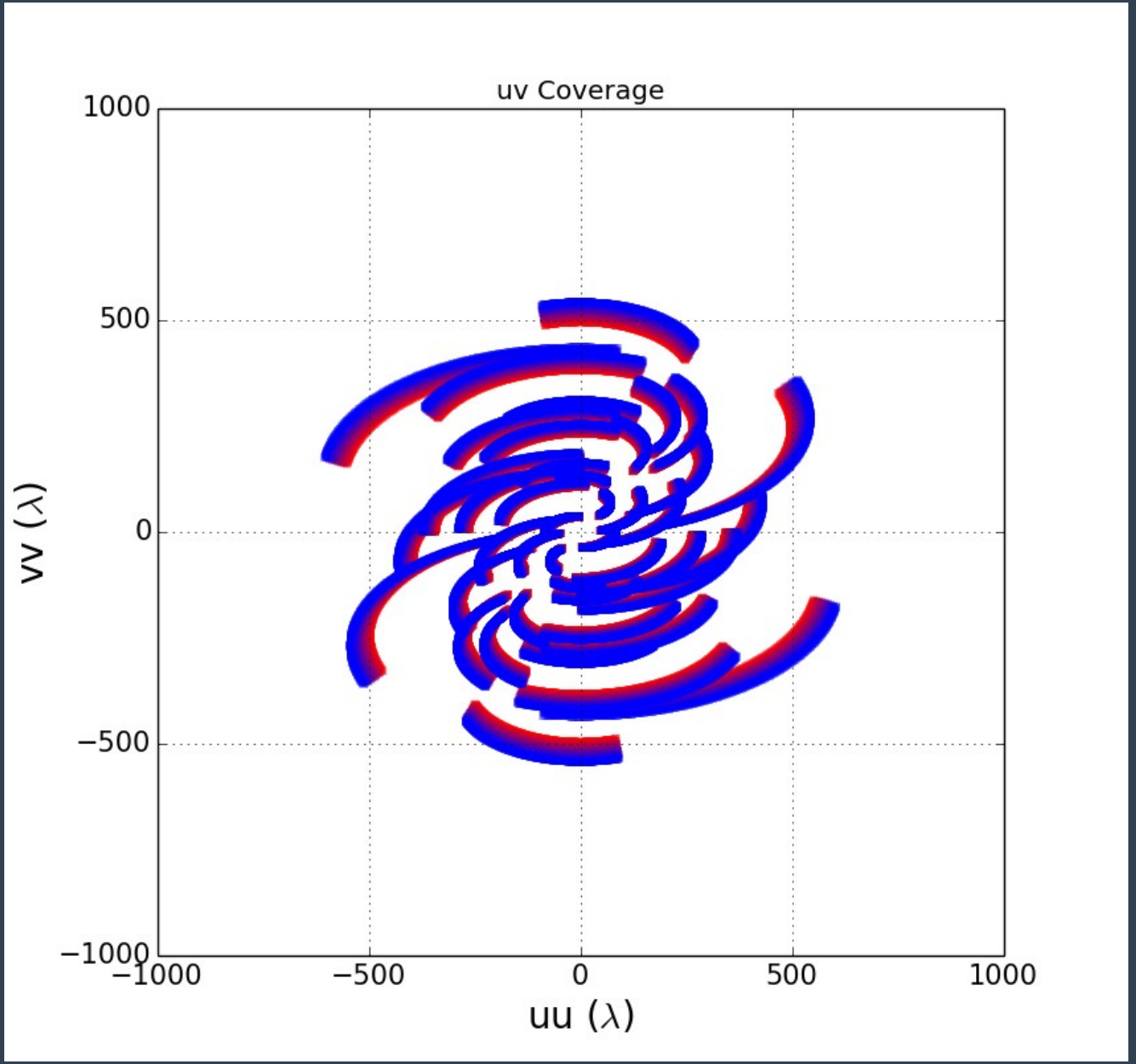




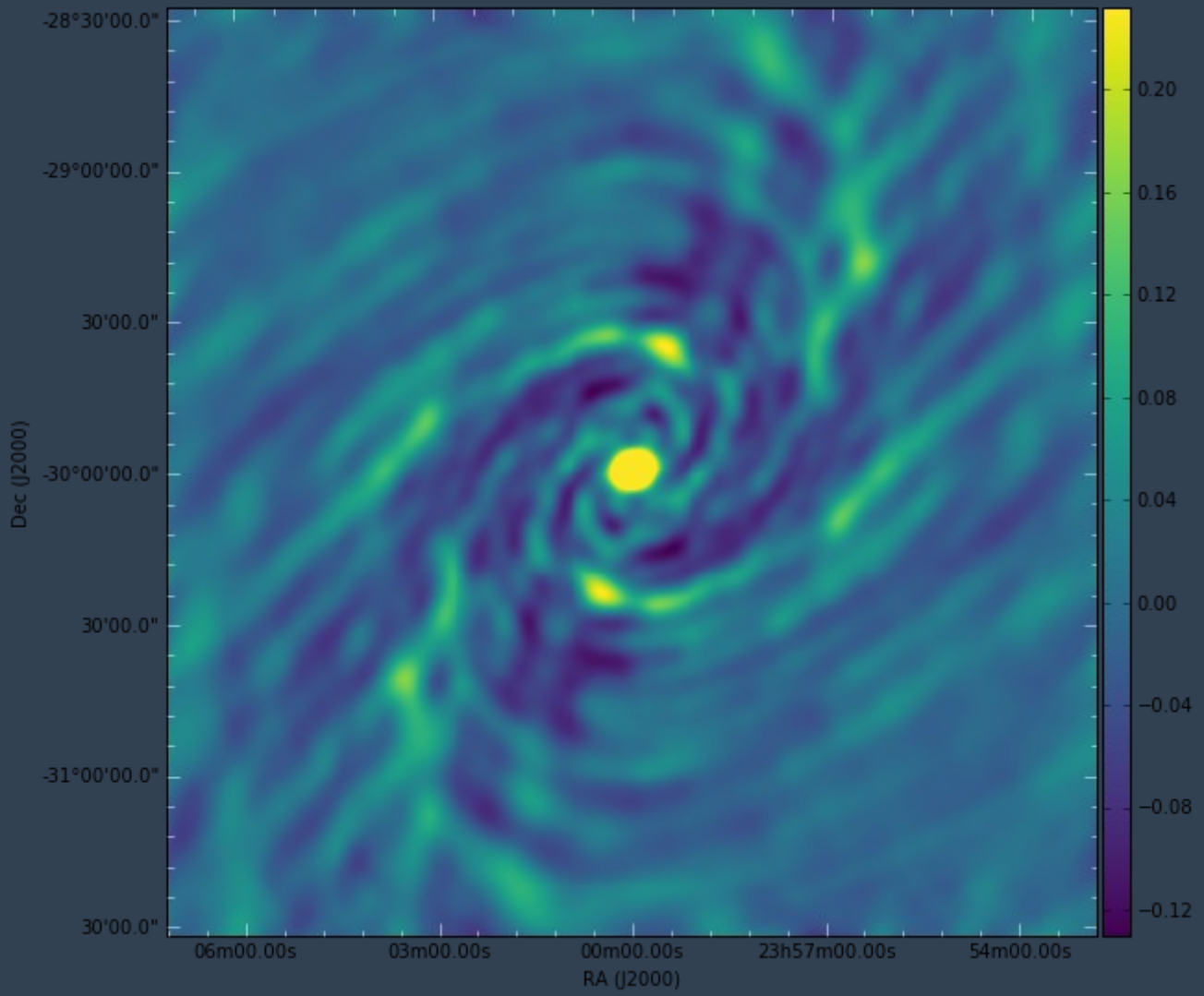
Hubble Telescope Image

KAT-7 Array

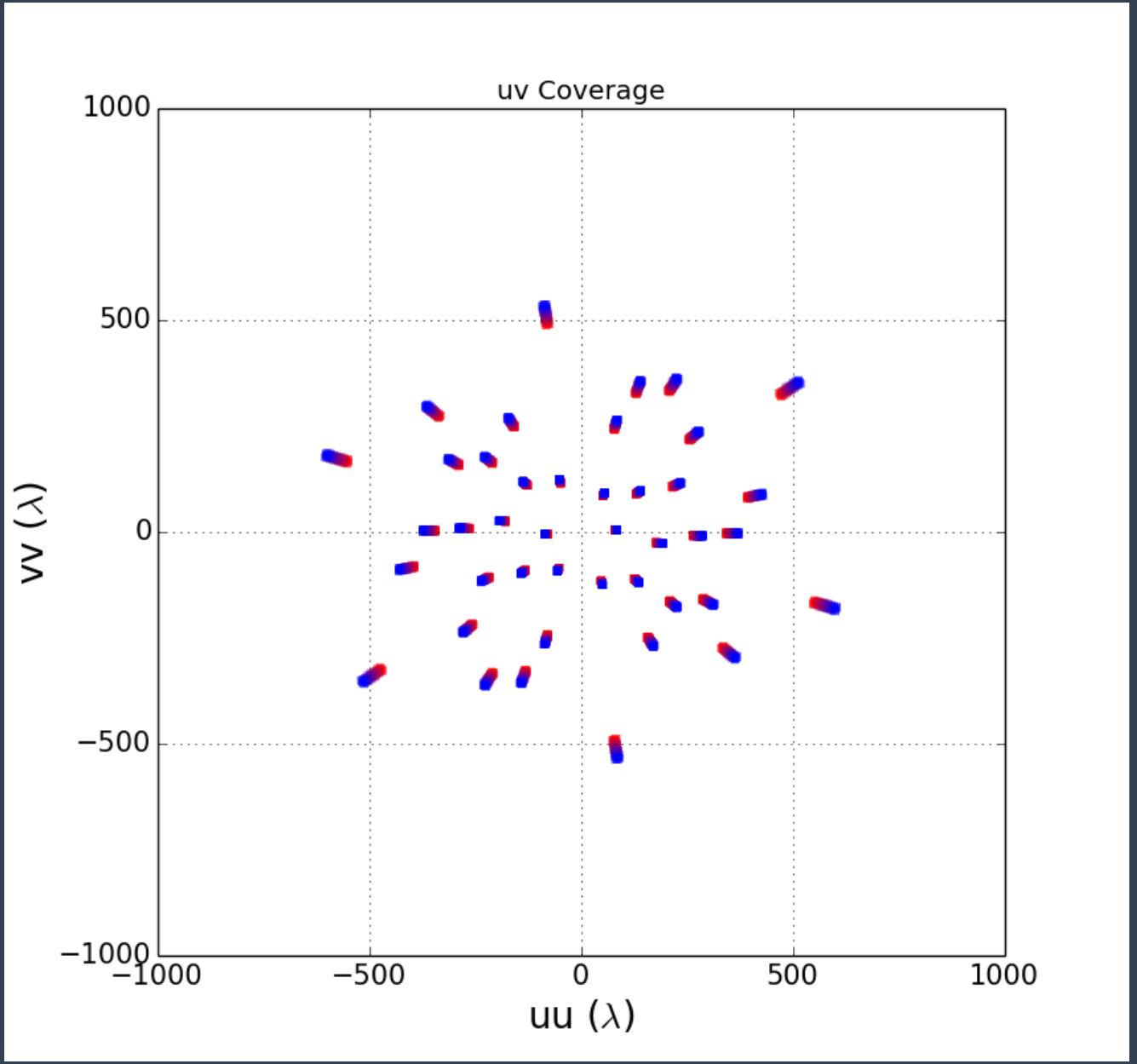
UV Coverage: 6 hours, 100MHz, -30 degrees



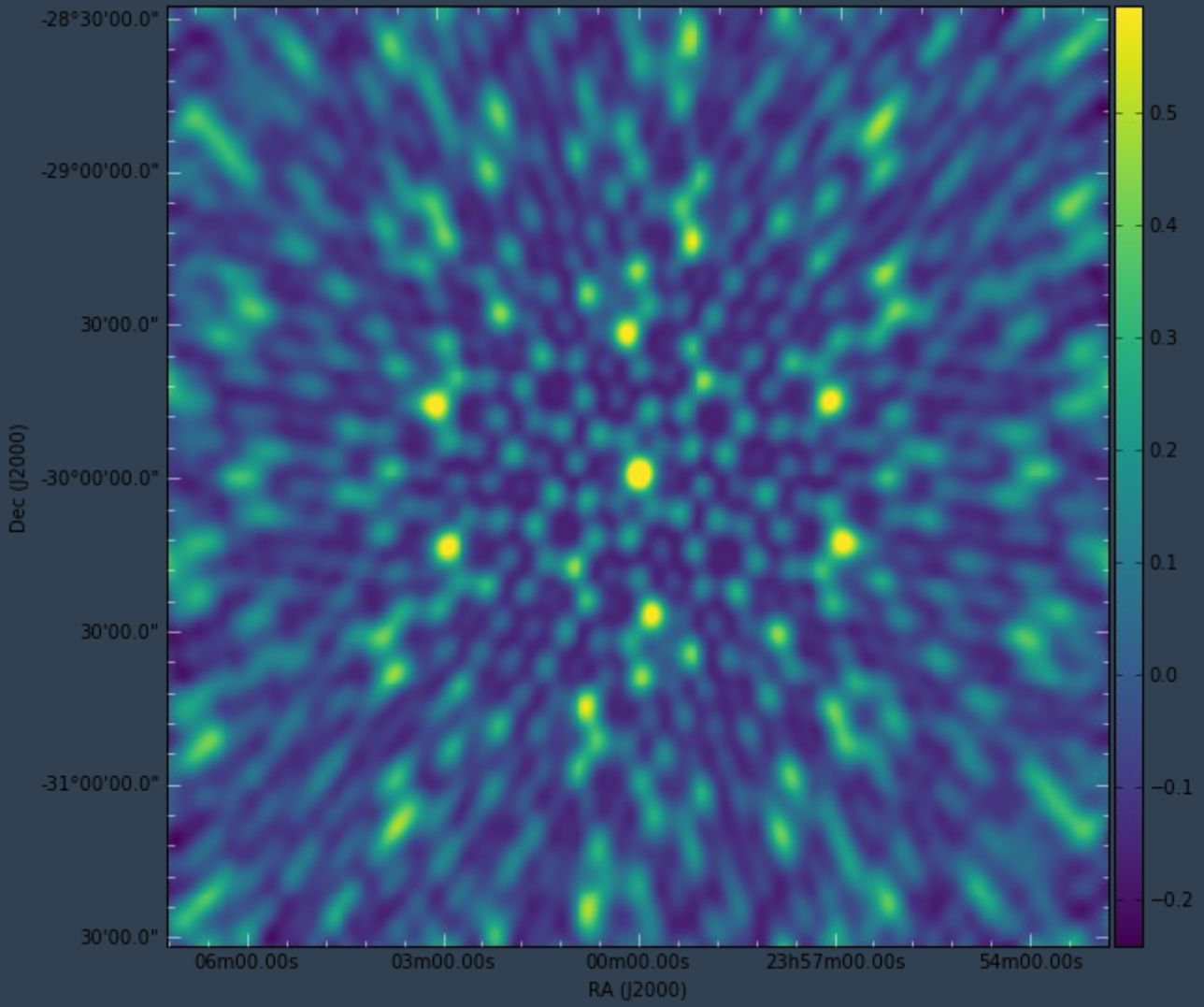
PSF: 6 hours, 100MHz, -30 degrees



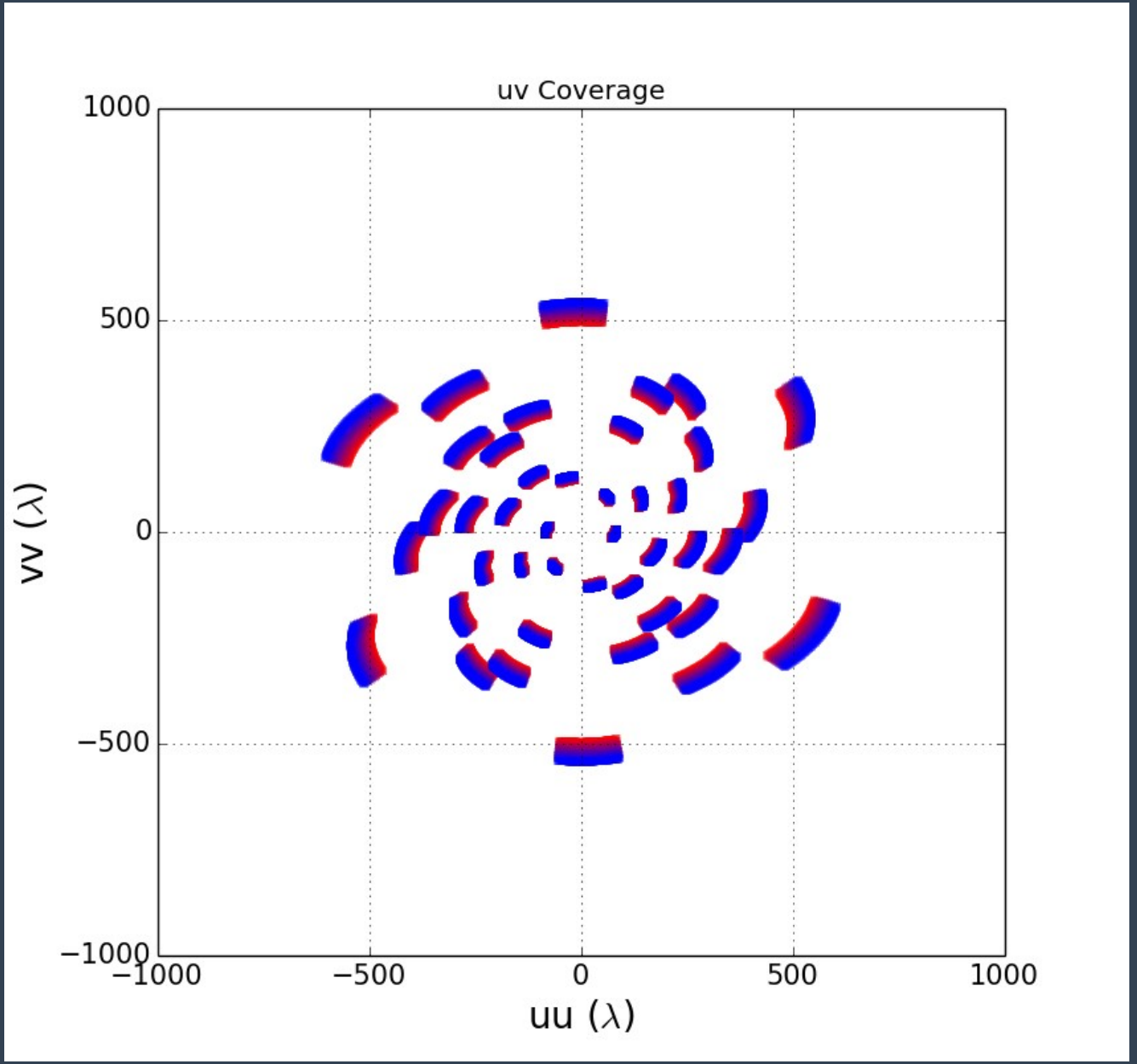
UV Coverage: 10 minutes, 100MHz, -30 degrees



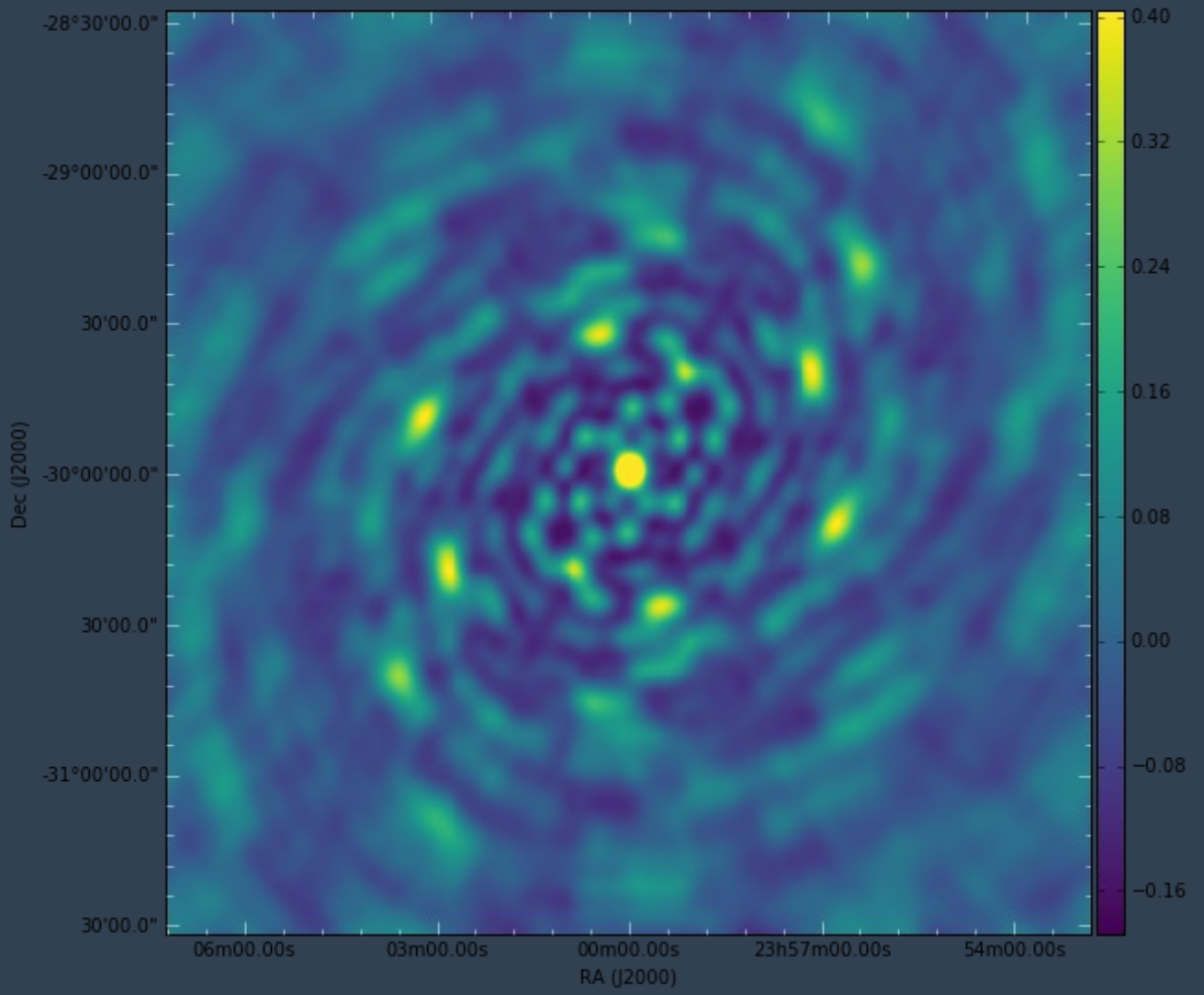
PSF: 10 minutes, 100MHz, -30 degrees



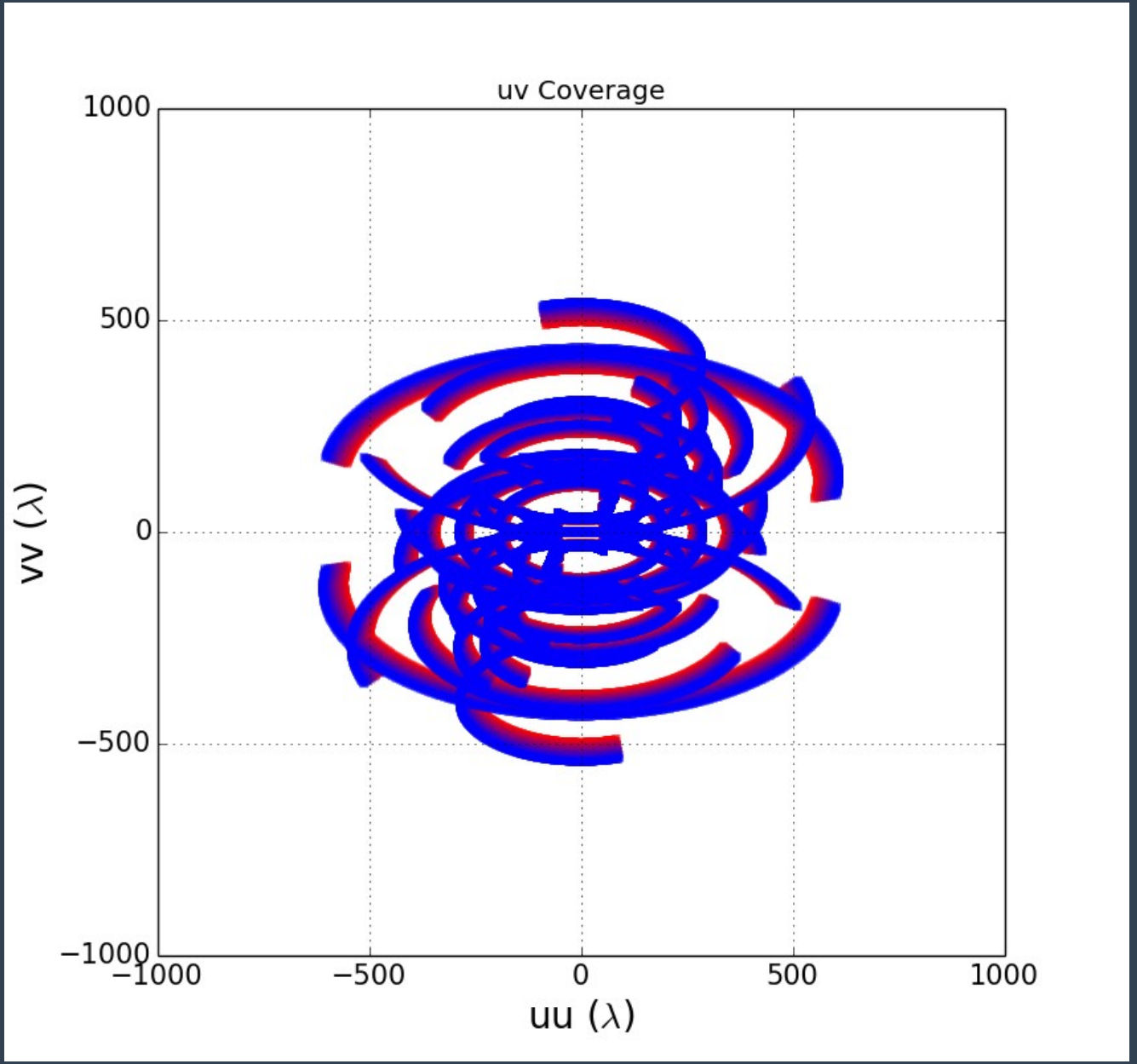
UV Coverage: 2 hours, 100MHz, -30 degrees



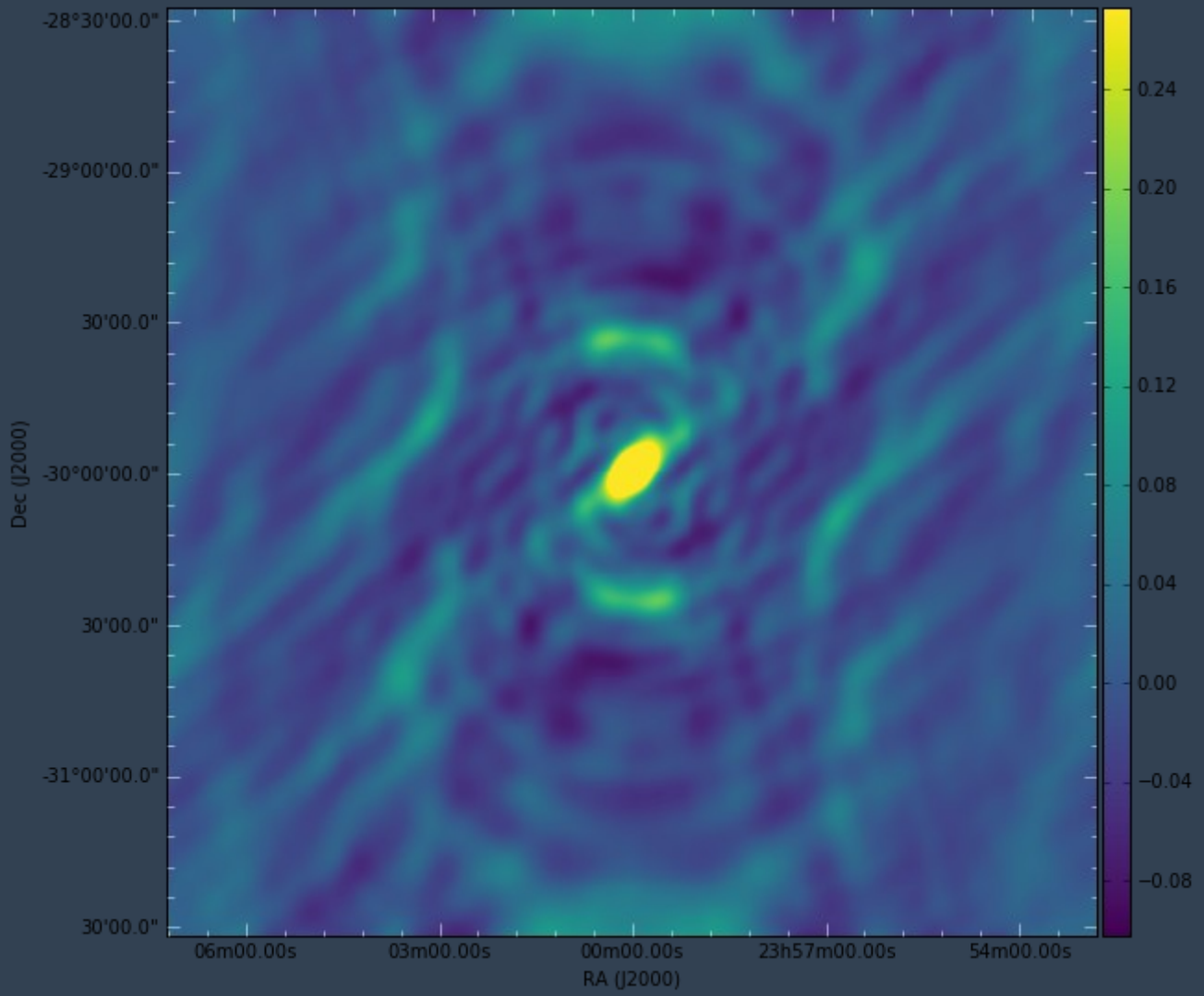
PSF: 2 hours, 100MHz, -30 degrees



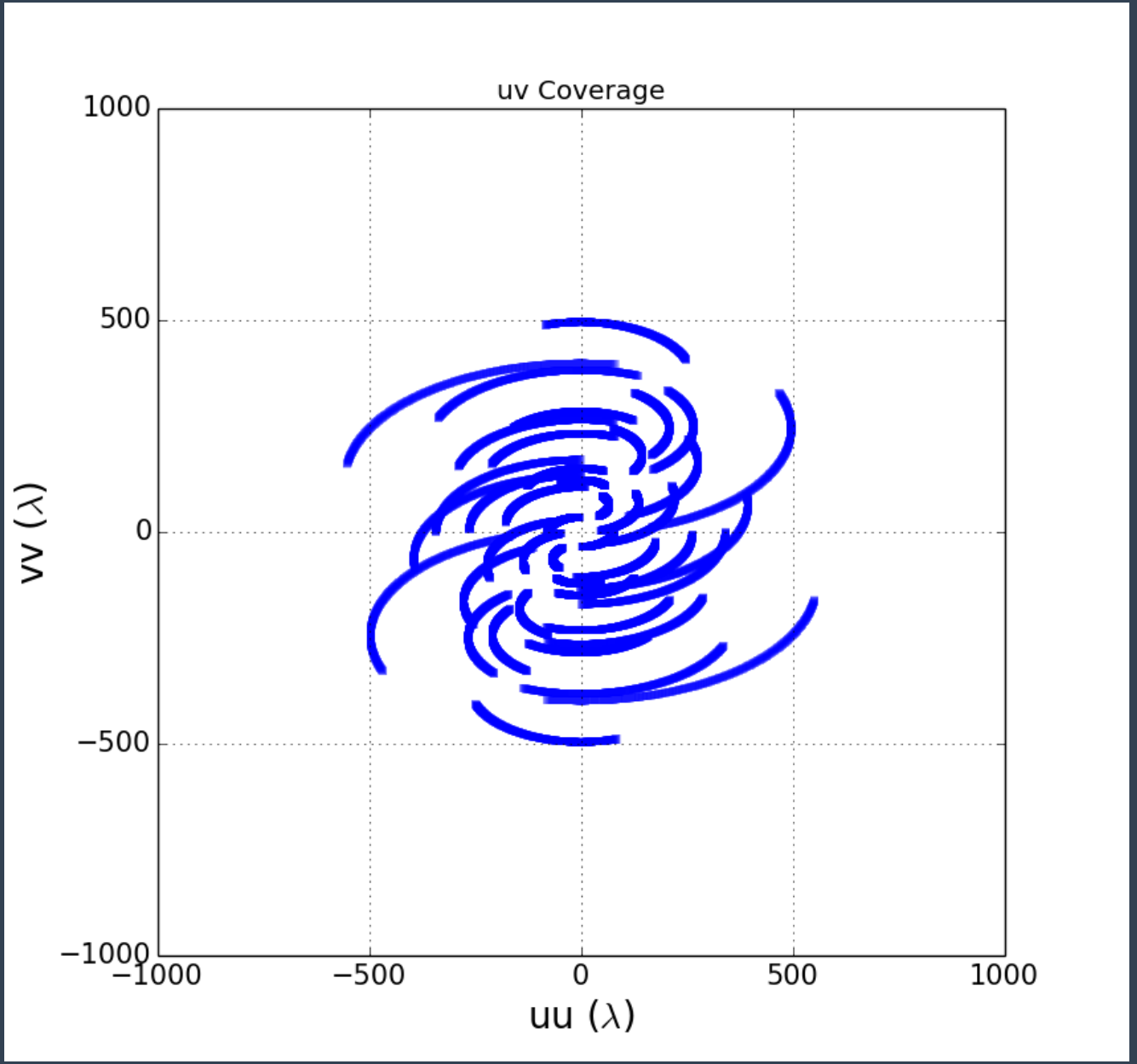
UV Coverage: 12 hours, 100MHz, -30 degrees



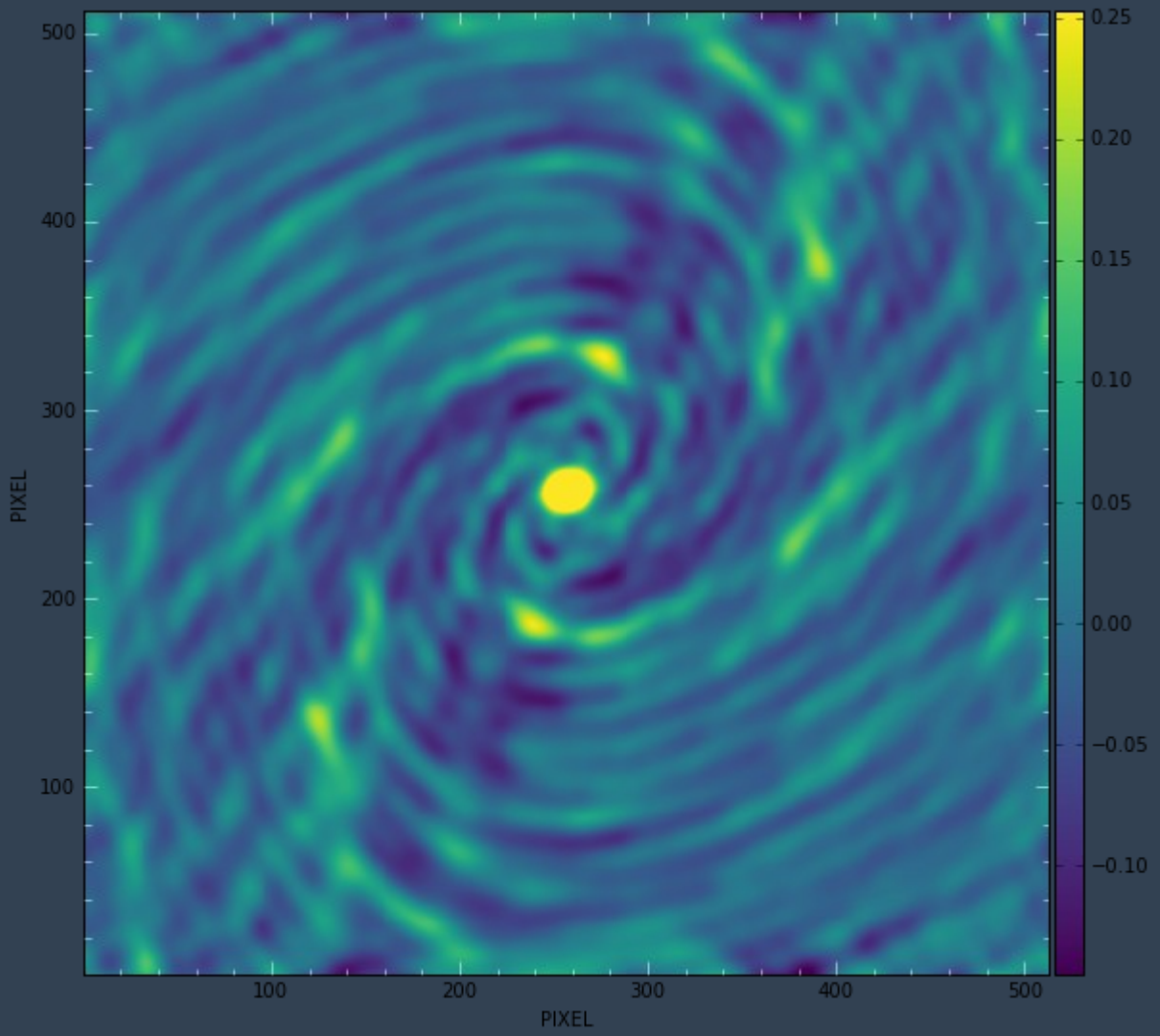
PSF: 12 hours, 100MHz, -30 degrees



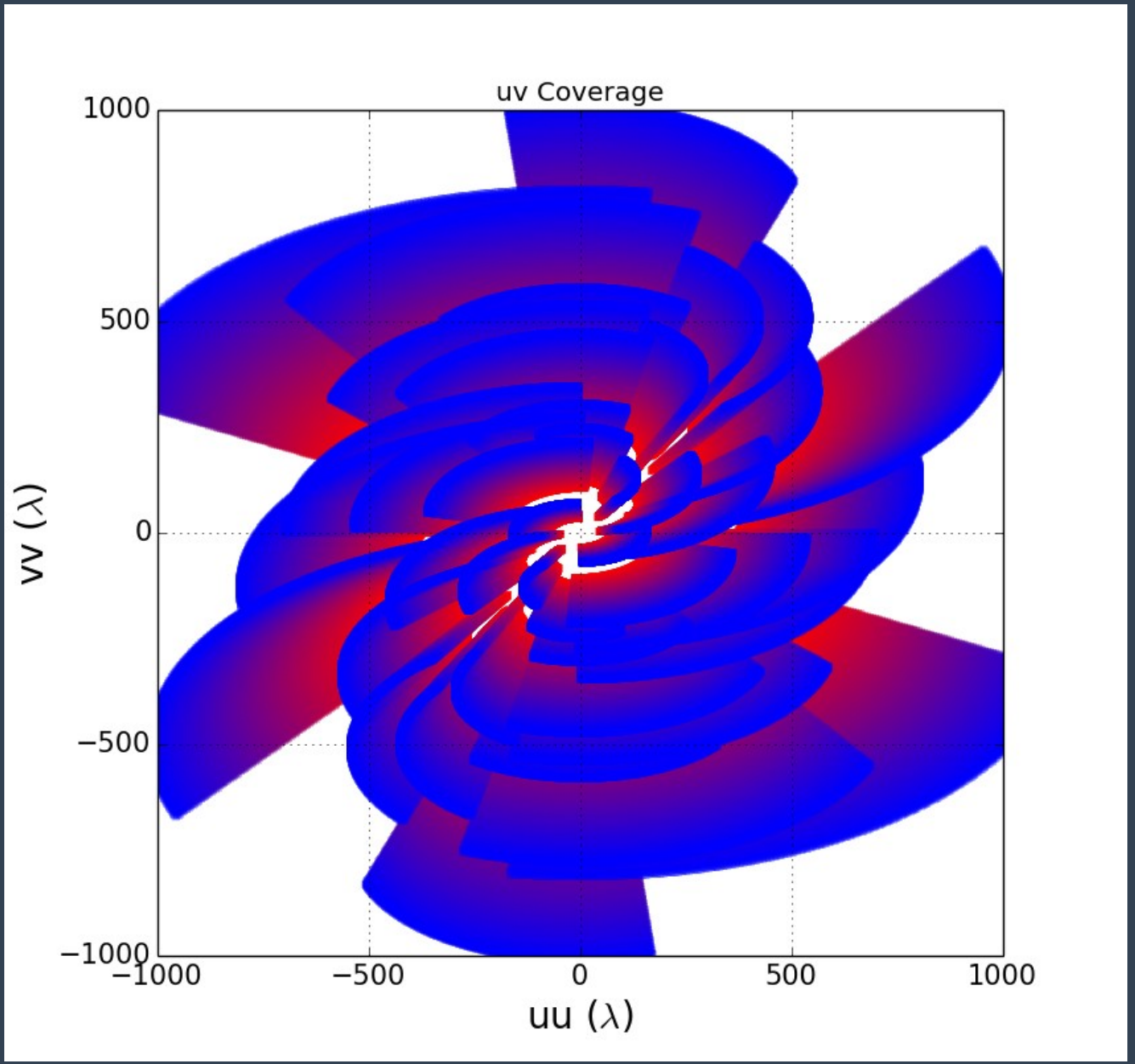
UV Coverage: 6 hours, 10 MHz, -30 degrees



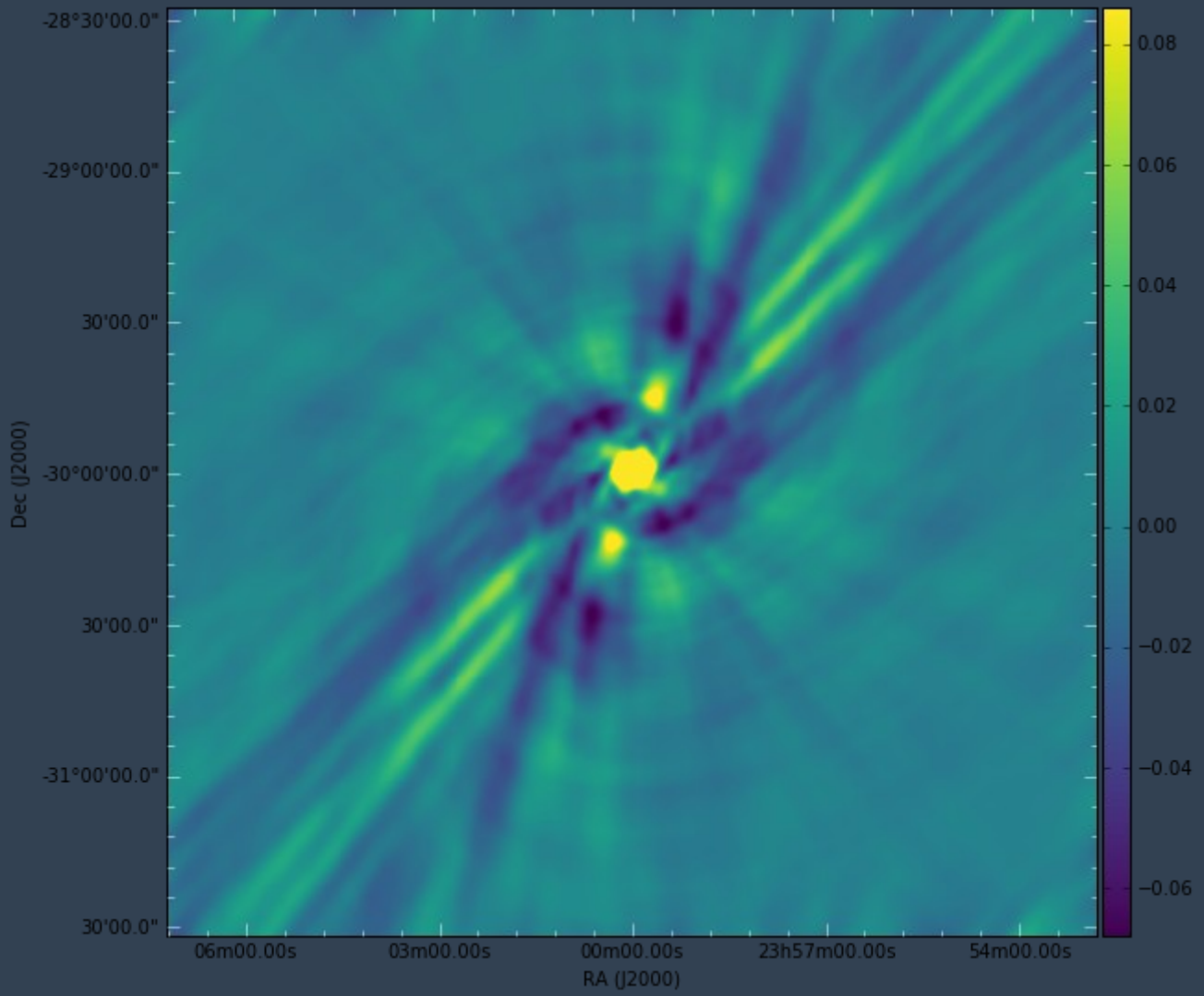
PSF: 6 hours, 10 MHz, -30 degrees



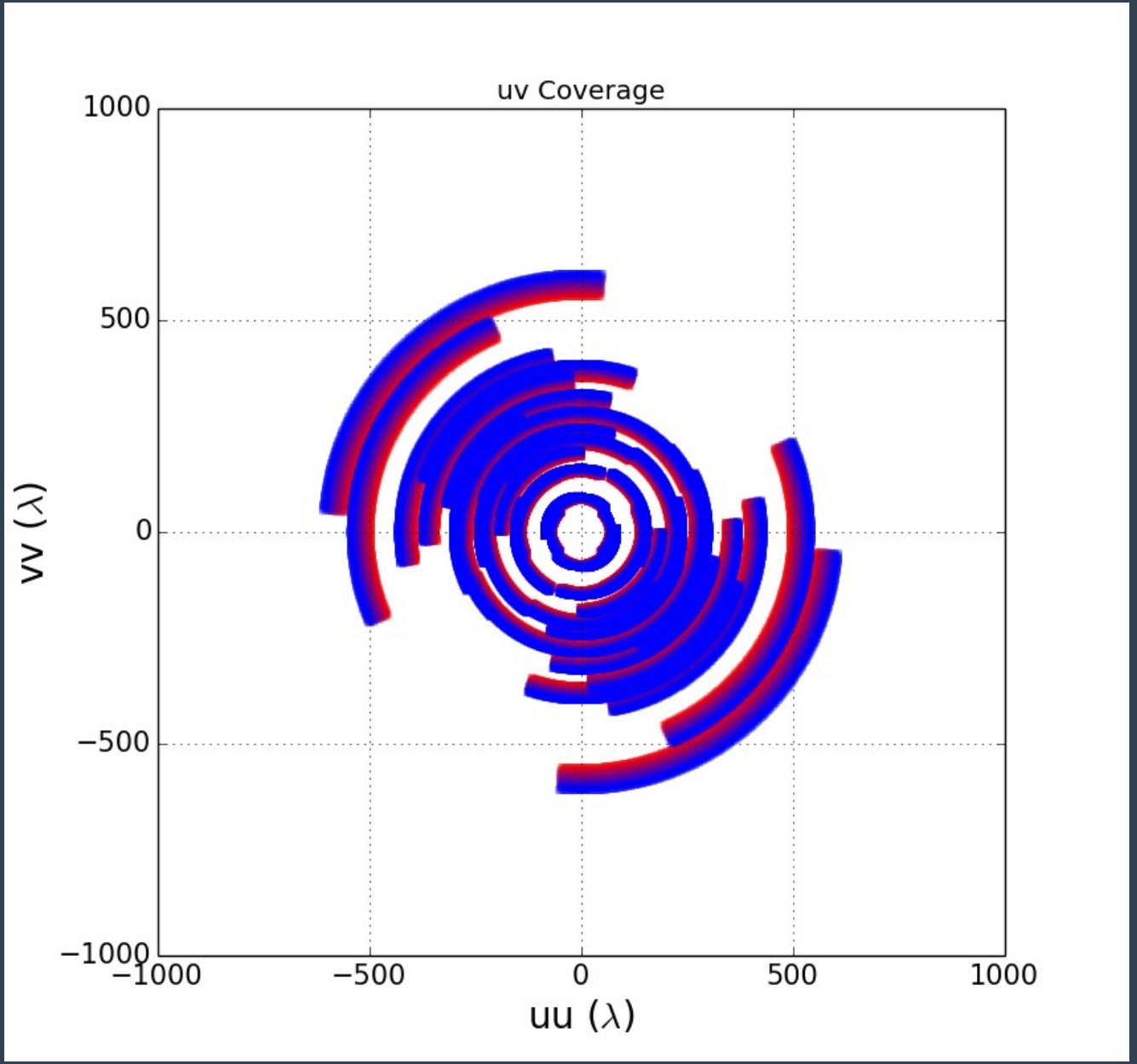
UV Coverage: 6 hours, 1 GHz, -30 degrees



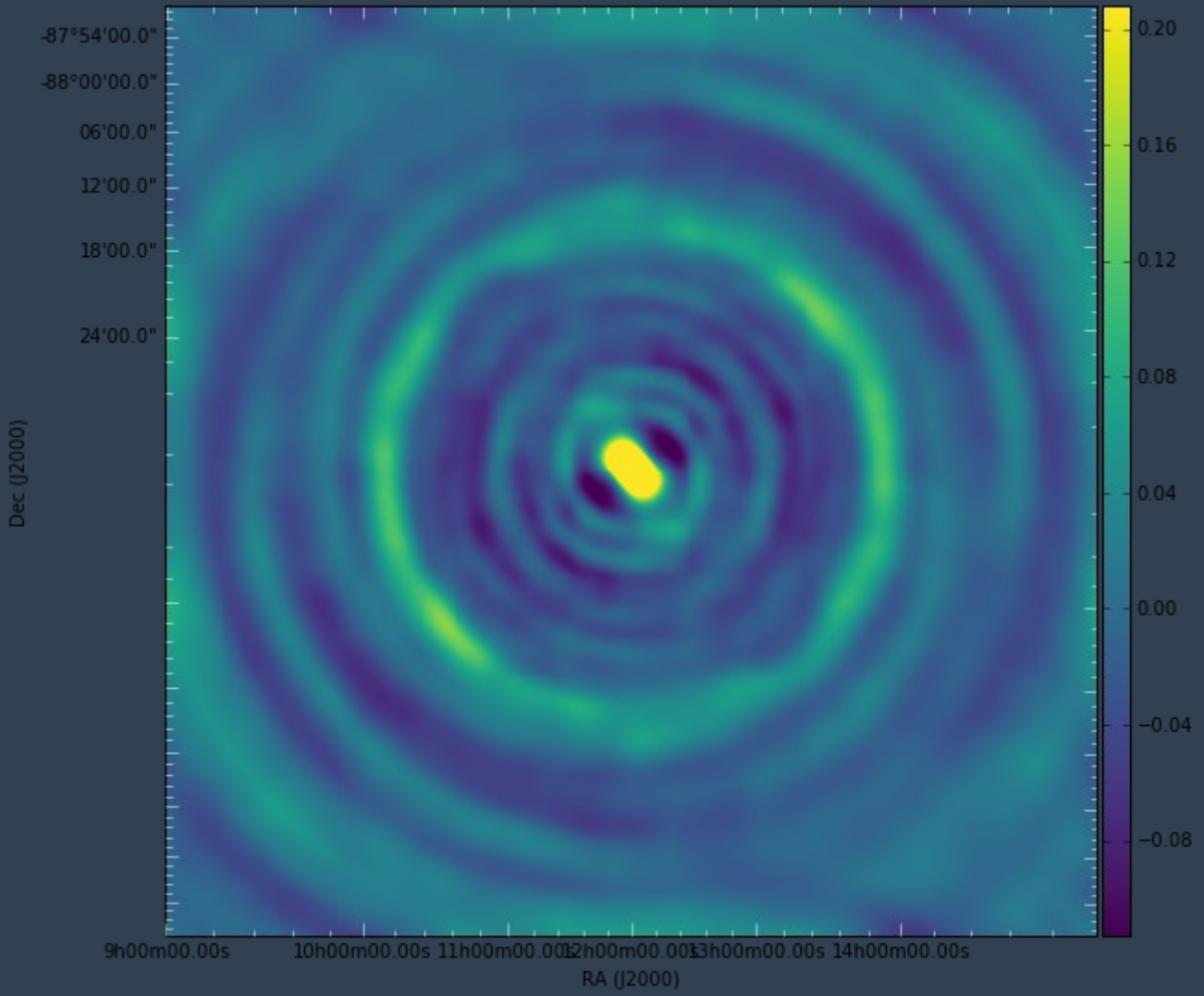
PSF: 6 hours, 1 GHz, -30 degrees



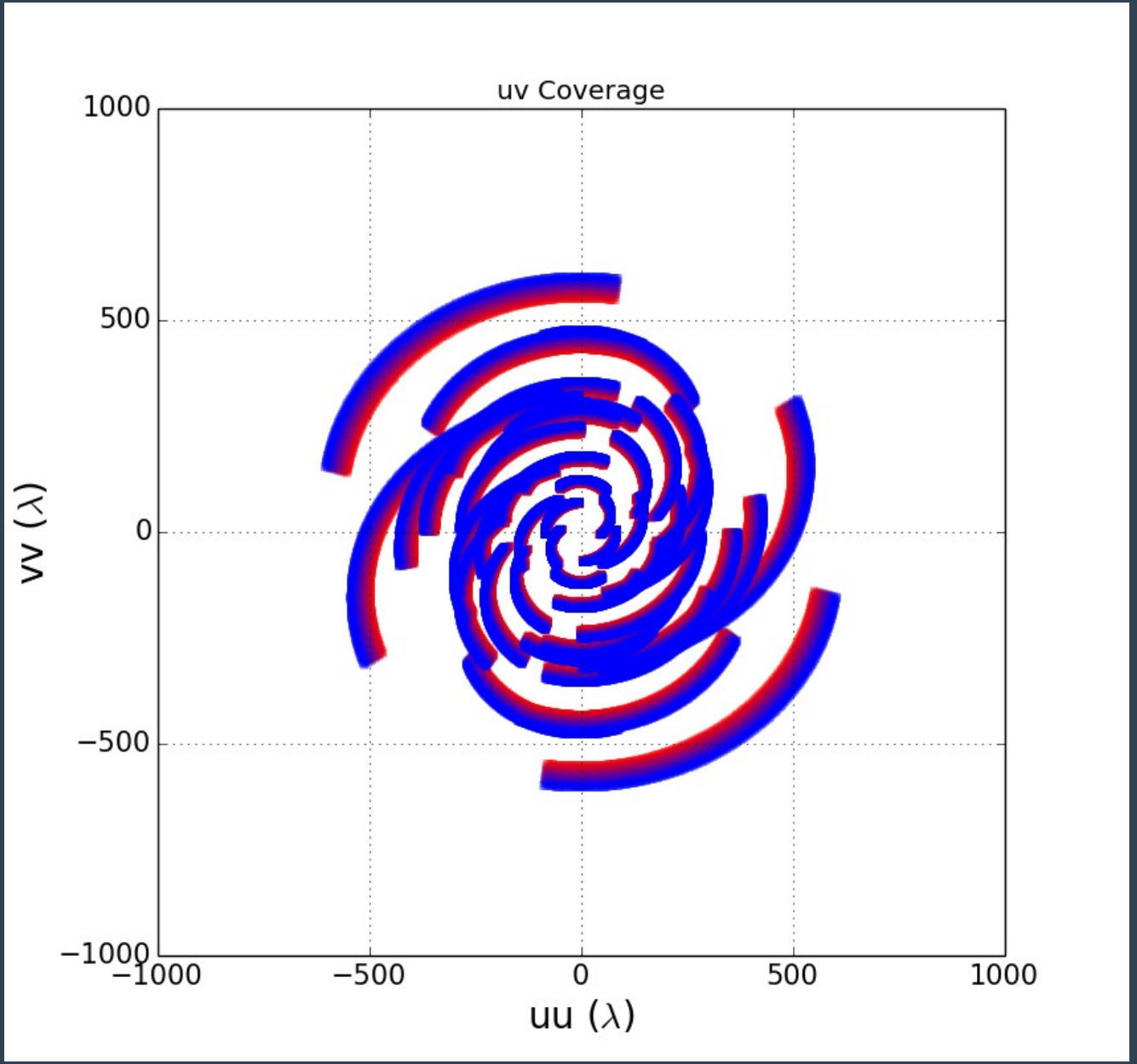
UV Coverage: 6 hours, 100 MHz, -90 degrees



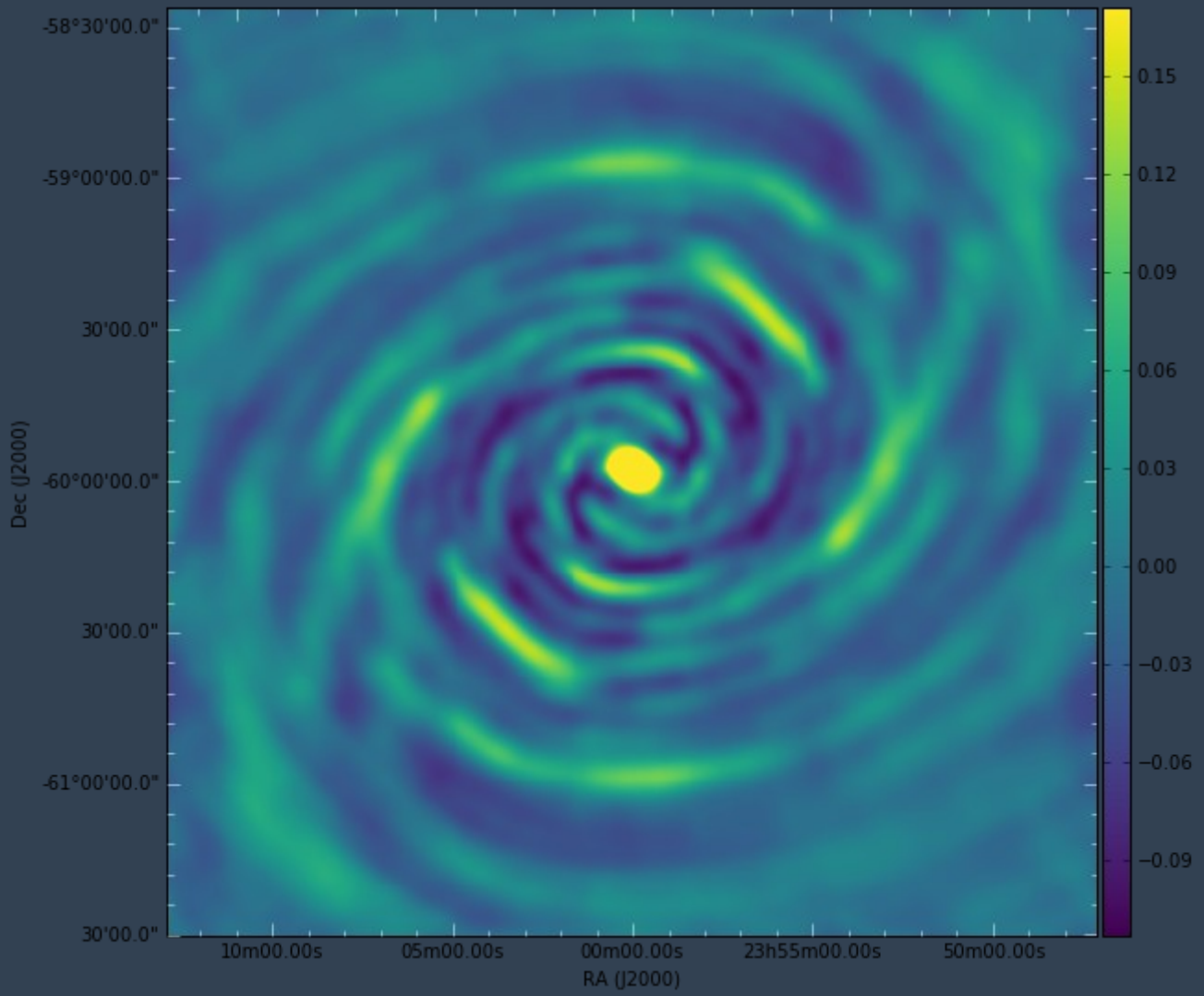
PSF: 6 hours, 100 MHz, -90 degrees



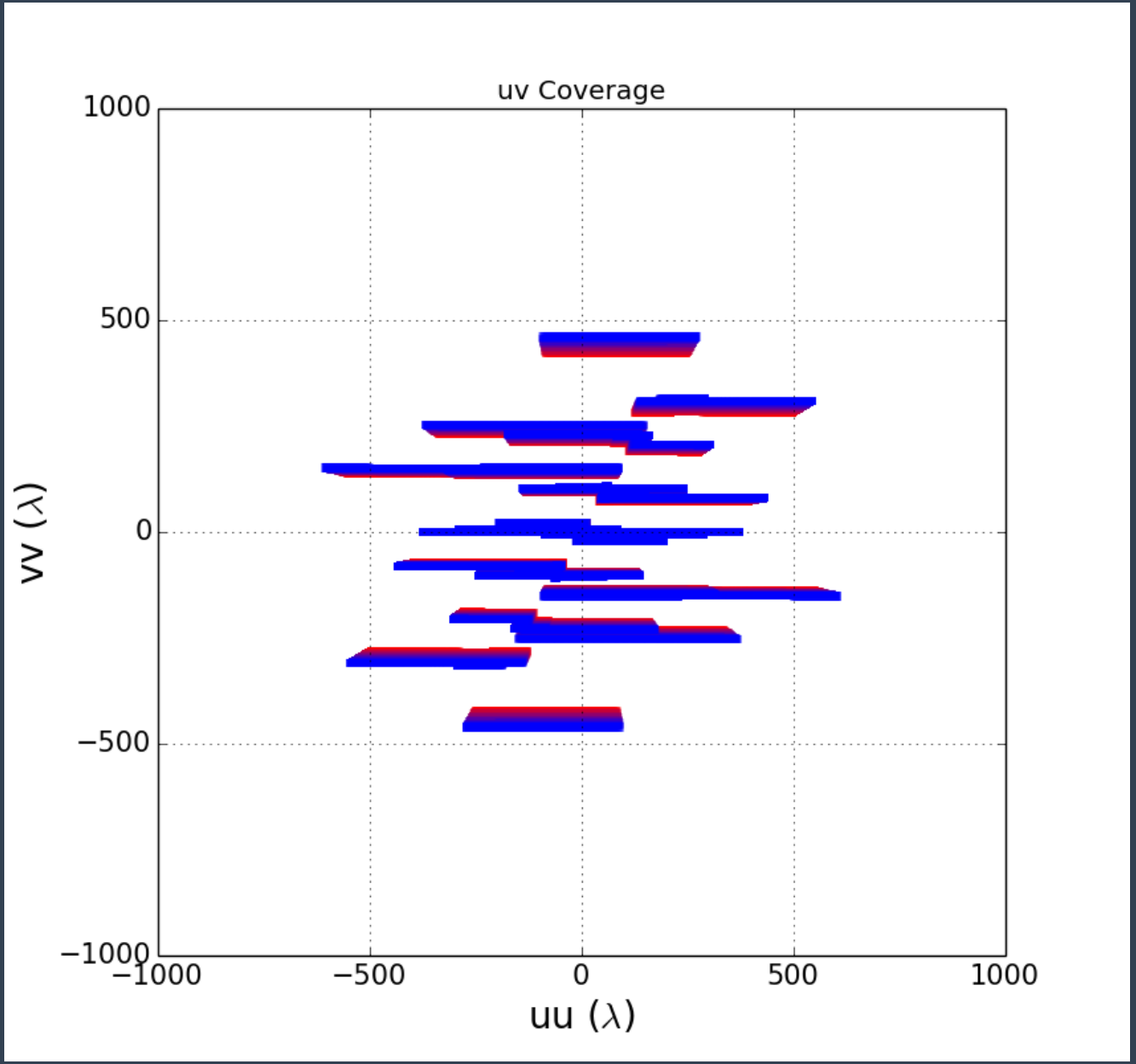
UV Coverage: 6 hours, 100 MHz, -60 degrees



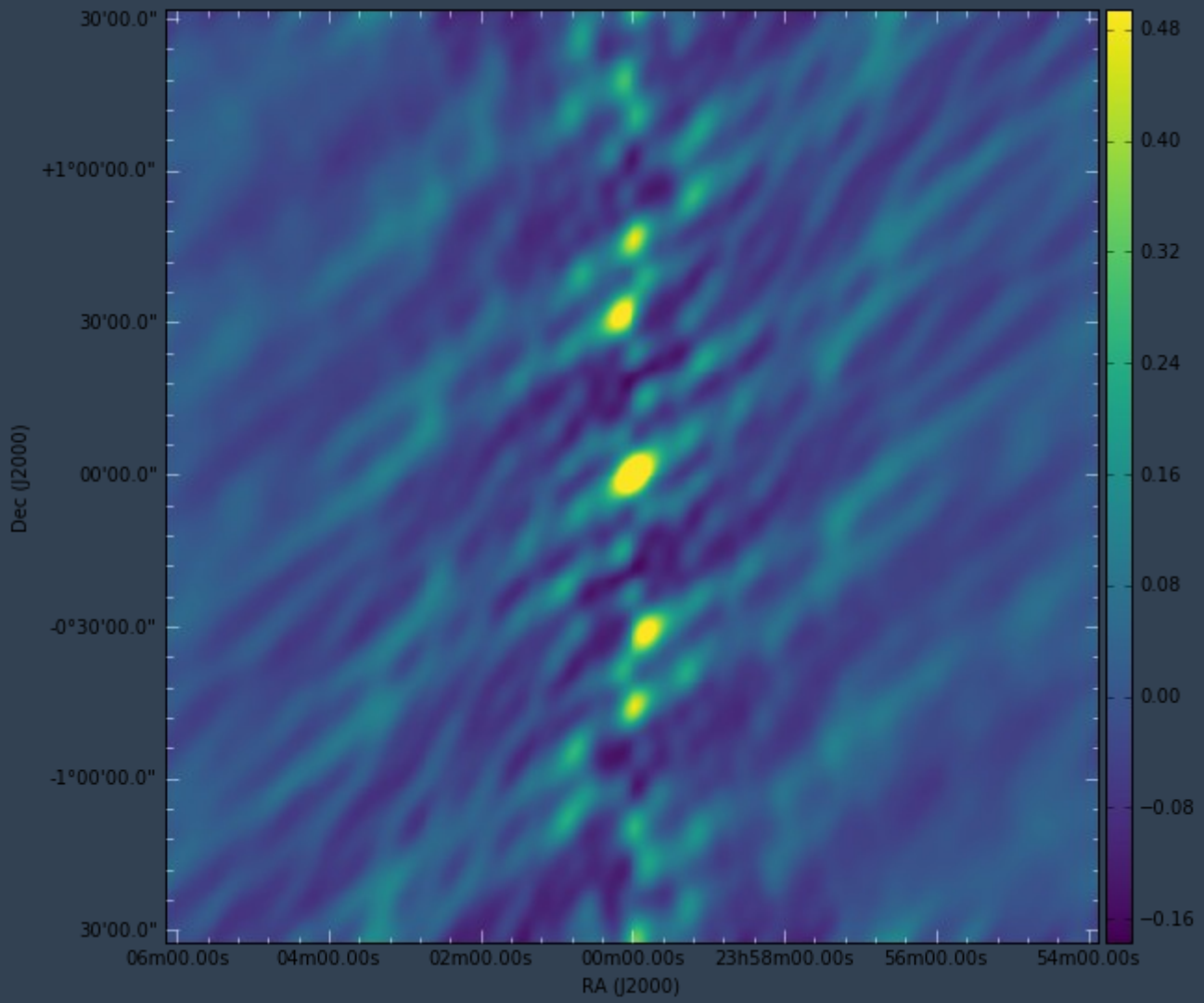
PSF: 6 hours, 100 MHz, -60 degrees



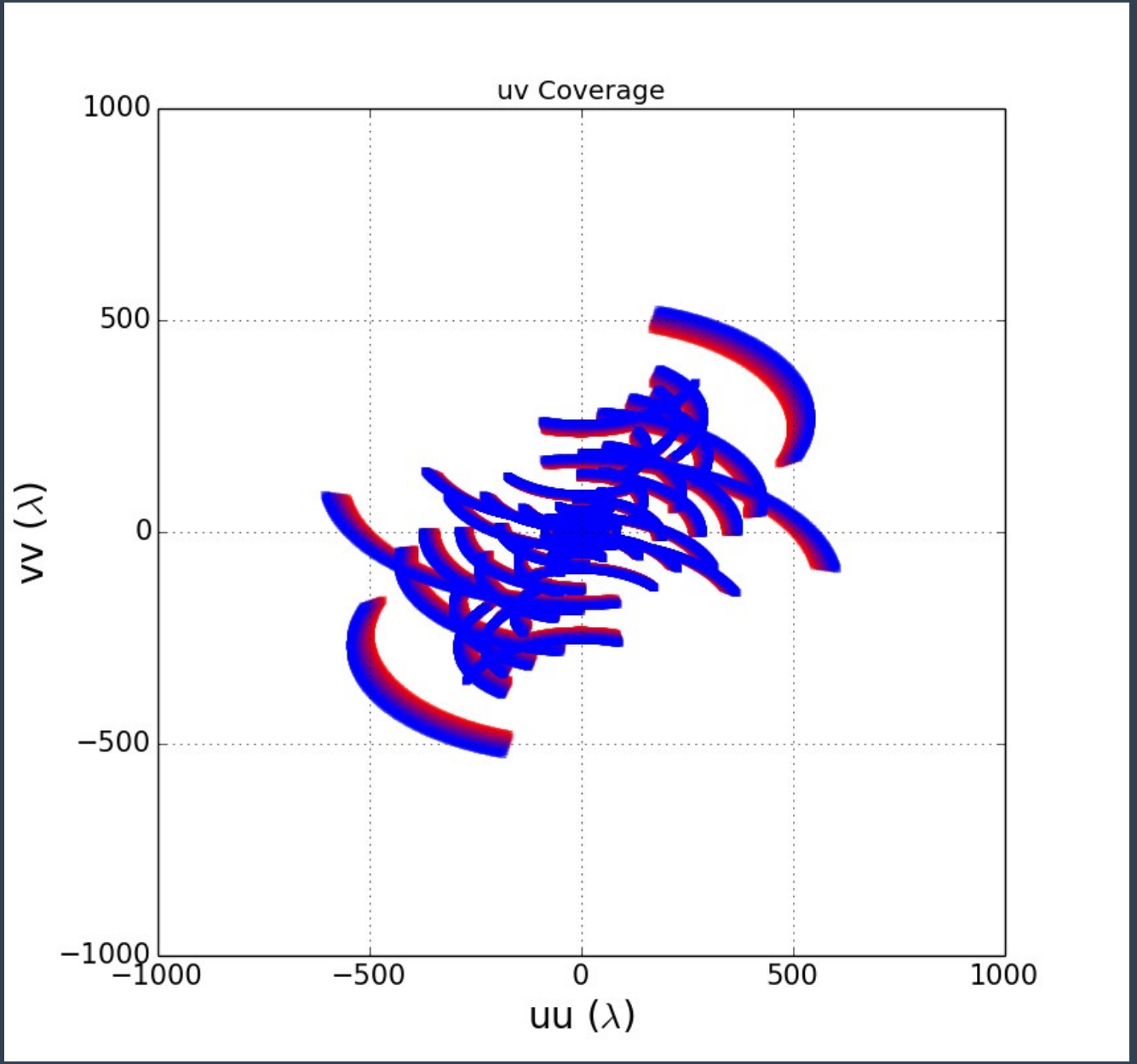
UV Coverage: 6 hours, 100 MHz, 0 degrees



PSF: 6 hours, 100 MHz, 0 degrees



UV Coverage: 6 hours, 100 MHz, +30 degrees



PSF: 6 hours, 100 MHz, +30 degrees